(10 pts) 1) Find an equation of the line perpendicular to $2x + y + 3 = 0$ and passing through the point $(-1, 1)$.

Equation of given line can be written as $y = -2x - 3$.

Line $L$ to given line has slope $\frac{1}{2}$.

Since line $L$ passes through $(-1, 1)$ it has

$$y - 1 = \frac{1}{2} (x + 1)$$

$$y = \frac{1}{2} x + \frac{3}{2}$$

$$2y - x - 3 = 0$$

ANSWER: $y = \frac{1}{2} x + \frac{3}{2}$  $2y - x - 3 = 0$

(10 pts) 2) Sketch the graph of $y = x^2 - 2x + 2$ without plotting points. State the steps you used to get your graph from the graph of a simple function.

\[ y = (x-1)^2 + 1 \]

To get graph, translate graph of $y = x^2$ one unit to the right and one unit up.
(7 pts) 3) (a) Find a formula for the inverse of the function

\[ f(x) = \frac{1 + 2x}{3 + 4x} \quad x \neq -\frac{3}{4} \]

(b) What is the domain of the inverse function?

Solve \( y = \frac{1 + 2x}{3 + 4x} \) for \( x \) and give \( f^{-1}(y) \)

\[ y(3+4x) = 1+2x \]
\[ 3y - 1 = (-4y+2)x \]
\[ x = \frac{3y-1}{2(-2y+1)} \]
\[ x = f^{-1}(y) \]

Relabeling \( x \) as the independent variable gives

\[ y = \frac{3x-1}{2(-2x+1)} \]
\[ y \neq \frac{1}{2} \]

**Inverse** \[ y = \frac{3x-1}{2(-2x+1)} \] \[ y = \frac{3x-1}{2-4x} \] \[ \text{Domain} \quad y \neq \frac{1}{2} \]

(10 pts) 4) Let

\[ f(x) = \begin{cases} 
  x + c & \text{if } x \leq 3 \\
  cx^2 - 2 & \text{if } x > 3 
\end{cases} \]

For what value of \( c \), if any, is the function continuous at \( x = 3 \)? Why?

For \( f(x) \) to be continuous at \( x = 3 \), we require \( f(3) = \lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x) \)

\[ \lim_{x \to 3^+} f(x) = 3 + c \quad \text{and} \quad \lim_{x \to 3^-} f(x) = 9c - 2 \]

We get that \( c \) must satisfy

\[ 3 + c = 9c - 2 \]
\[ 5 = 8c \]
\[ c = \frac{5}{8} \]

**Answer** \[ \frac{5}{8} \]
5) Find the following limits or show that they do not exist.

(a) \( \lim_{x \to \infty} \frac{7x^3 + 2x + 100}{8x^3 + 12x^2 + 2x + 1} \)

\[
\frac{7x^3 + 2x + 100}{8x^3 + 12x^2 + 2x + 1} = \frac{7}{8} \left( 1 + \frac{2}{x^2} + \frac{100}{x^3} \right)
\]

\[
\lim_{x \to \infty} \frac{7}{8} = \frac{7}{8}
\]

**Answer: \( \frac{7}{8} \)**

(b) \( \lim_{x \to -4} \frac{x^2 + x - 12}{x + 4} \)

\[
\frac{x^2 + x - 12}{x + 4} = \frac{(x + 4)(x - 3)}{x + 4} = (x - 3) \quad \text{as} \quad x \to -4
\]

\[
\lim_{x \to -4} (x - 3) = -7
\]

**Answer: -7**

(c) \( \lim_{x \to 0} |x| \cos \frac{\pi}{x^2} \)

\[
-1 \leq \cos \frac{\pi}{x^2} \leq 1
\]

\[
-|x| \leq |x| \cos \frac{\pi}{x^2} \leq |x|
\]

Since \( |x| \to 0 \) as \( x \to 0 \), by the Squeeze Theorem, \( \lim_{x \to 0} |x| \cos \frac{\pi}{x^2} = 0 \)

**Answer: 0**
NOTE: IN PROBLEMS 6 AND 7 YOU CANNOT USE THE DIFFERENTIATION RULES OF CHAP. 3.

(10 pts) 6) Find the slope of the tangent line to the graph of

\[ y = \frac{1}{x + 1}, \quad x \neq -1 \]

at the point with \( x \) coordinate = 2.

\[
\text{Slope of tangent line} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{1}{2 + h + 1} - \frac{1}{2 + 1} \right] = \lim_{h \to 0} \frac{(2 + 1) - (2 + 1 + h)}{(2 + 1)(2 + 1 + h)} \cdot \frac{1}{h}
\]

\[
= \lim_{h \to 0} \frac{-h}{(2 + 1)(2 + h + 1)} \cdot \frac{1}{h} = \frac{-1}{(3)^2} = -\frac{1}{9}
\]

ANSWER \[-\frac{1}{9}\]

7) Let \( f(x) = \sqrt{2 + x} \)

(5 pts) (a) What is the domain of \( f \)?

(10 pts) (b) Find \( f'(x) \).

\[
f'(x) = \lim_{h \to 0} \frac{\sqrt{2 + (x+h)} - \sqrt{2 + x}}{h}
\]

\[
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{\sqrt{2 + (x+h)} - \sqrt{2 + x}}{h} \right] = \lim_{h \to 0} \frac{1}{h} \left[ \frac{2 + (x+h) - (2 + x)}{\sqrt{2 + (x+h)} + \sqrt{2 + x}} \right] = \lim_{h \to 0} \frac{1}{h} \left[ \frac{h}{\sqrt{2 + x} + \sqrt{2 + x}} \right] = \frac{1}{2 \sqrt{2 + x}}
\]

DOMAIN \[x \geq -2\]

\[f'(x) = \frac{1}{2 \sqrt{2 + x}}\]
8) Show that the equation \( x^5 - 2x^2 + 1 = 0 \) has at least one negative root. What theorem justifies your conclusion?

By intermediate value theorem if \( f(x) = x^5 - 2x^2 + 1 \)

It is positive at some integer \( k \) and negative at \( k-1 \)

(or vice versa) then by the intermediate value theorem

there is a point in the interval \((k-1, k)\) at which

\( f(x) = 0 \), say \( x = a \). Thus \( a \) is root of \( x^5 - 2x^2 + 1 = 0 \).

Make table of values

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
</tbody>
</table>

By intermediate value theorem \( x^5 - 2x^2 + 1 \) has a root between

-1 and 0.