Volumes of Revolution

1. If the region in Fig. 1 is revolved about

   (a) the $x$-axis:  $V = \pi \int_a^b [(y_2)^2 - (y_1)^2] \, dx$  
   (b) the $y$-axis:  $V = 2\pi \int_a^b x(y_2 - y_1) \, dx$

2. If the region in Fig. 2 is revolved about

   (a) the $x$-axis:  $V = 2\pi \int_c^d y(x_2 - x_1) \, dy$  
   (b) the $y$-axis:  $V = \pi \int_c^d [(x_2)^2 - (x_1)^2] \, dy$

Moments and Centroids

1. For the region in Fig. 1:  $M_x = \frac{1}{2} \int_a^b [(y_2)^2 - (y_1)^2] \, dx$,  
   $M_y = \int_a^b x(y_2 - y_1) \, dx$

2. For the region in Fig. 2:  $M_x = \int_c^d y(x_2 - x_1) \, dy$,  
   $M_y = \frac{1}{2} \int_c^d [(x_2)^2 - (x_1)^2] \, dy$

The centroid of a plane region having area $A$ is located at $(\bar{x}, \bar{y})$, where

\[
\bar{x} = \frac{M_y}{A}, \quad \bar{y} = \frac{M_x}{A}.
\]

Mean and Root Mean Square

\[
f_{av} = \frac{1}{b-a} \int_a^b f(x) \, dx, \quad f_{rms} = \left\{ \frac{1}{b-a} \int_a^b [f(x)]^2 \, dx \right\}^{1/2}
\]

Work

The work done in moving an object along the $x$-axis from $x = a$ to $x = b$ by a force $f(x)$ is

\[
W = \int_a^b f(x) \, dx,
\]

Fluid Pressure

The pressure $p$ of a fluid in an open container, at a point $y$ units below the surface, is $p = wgy$, where $w$ is the weight per unit volume of the fluid. If $\rho$ is the density of the fluid (mass/unit volume), and $g$ is the gravitational constant, then $w = \rho g$, so $p = \rho gy$. 