MA 224 FORMULAS

THE SECOND DERIVATIVE TEST
Suppose \( f \) is a function of two variables \( x \) and \( y \), and that all the second-order partial derivatives are continuous. Let
\[
D = f_{xx}f_{yy} - (f_{xy})^2
\]
and suppose \((a, b)\) is a critical point of \( f \).
1. If \( D(a, b) < 0 \), then \( f \) has a saddle point at \((a, b)\).
2. If \( D(a, b) > 0 \) and \( f_{xx}(a, b) < 0 \), then \( f \) has a relative maximum at \((a, b)\).
3. If \( D(a, b) > 0 \) and \( f_{xx}(a, b) > 0 \), then \( f \) has a relative minimum at \((a, b)\).
4. If \( D(a, b) = 0 \), the test is inconclusive.

LEAST-SQUARES LINE
The equation of the least-squares line for the \( n \) points \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\), is
\[
y = mx + b, \text{ where } m = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}, \quad b = \frac{\sum x^2 \sum y - \sum x \sum xy}{n \sum x^2 - (\sum x)^2}.
\]

TRAPEZOIDAL RULE
\[
\int_a^b f(x) dx \approx \frac{\Delta x}{2} \left[ f(x_1) + 2f(x_2) + \cdots + 2f(x_n) + f(x_{n+1}) \right],
\]
where \( x_{i+1} - x_i = \Delta x = \frac{b-a}{n}, \ x_1 = a, \ x_{n+1} = b \).

ERROR ESTIMATE FOR THE TRAPEZOIDAL RULE
If \( M \) is the maximum value of \(|f''(x)|\) on the interval \( a \leq x \leq b \), then
\[
|E_n| \leq \frac{M(b-a)^3}{12n^2}
\]

PROBABILITY
If \( f(x) \) is a probability density function, then
\[
\text{Expected Value (Mean)} = E = \int_{-\infty}^{\infty} xf(x) dx \\
\text{Variance} = V = \int_{-\infty}^{\infty} x^2 f(x) dx - E^2
\]

GEOMETRIC SERIES
If \(|r| < 1\), with \( r \neq 0 \), then
\[
\sum_{n=0}^{\infty} r^n = \frac{1}{1 - r}.
\]

TAYLOR SERIES
The Taylor series of \( f(x) \) about \( x = c \) is the power series \( \sum_{n=0}^{\infty} a_n(x-c)^n \), where
\[
a_n = \frac{f^{(n)}(c)}{n!}.
\]

Examples: (with \( c = 0 \))
\[
e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } -\infty < x < \infty; \quad \ln(1 + x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}, \text{ for } -1 < x \leq 1.
\]