Linear Systems

Consider

\[ X' = AX \]

where \( A \) is \( nxn \). If there are \( n \) linearly independent eigenvectors for \( A \), \( \{v(1), \ldots, v(n)\} \), then a fundamental set of solutions is given by \( \{\exp(s(i)*t)v(i), i=1, \ldots, n\} \). If \( n \) is large though this may be hard to find by hand. Here we show how to use Matlab to find the eigenvalues and eigenvectors.

Example

Let

\[
A = \\
\begin{bmatrix}
-1 & 0 & 2 \\
2 & 3 & 6 \\
-2 & 0 & -1
\end{bmatrix}
\]

Type:

\[
[\mathbf{E}, \mathbf{V}] = \text{eig}(A)
\]

We get

\[
\mathbf{E} = \\
\begin{bmatrix}
0 & 0.5000 & 0.5000 \\
1.0000 & 0.1000 - 0.7000i & 0.1000 + 0.7000i \\
0 & 0 + 0.5000i & 0 - 0.5000i
\end{bmatrix}
\]

\[
\mathbf{V} = \\
\begin{bmatrix}
3.0000 & 0 & 0 \\
0 & -1.0000 + 2.0000i & 0 \\
0 & 0 & -1.0000 - 2.0000i
\end{bmatrix}
\]

The diagonal matrix \( \mathbf{V} \) lists the eigenvalues of \( A \) and the corresponding columns of \( \mathbf{E} \) are the eigenvectors. Using this information we can write down 3 linearly independent real solutions (a fundamental set).

\[
X(1) = \exp(3t) \cdot [0, 1, 0]' \\
X(2) = \exp(-t) \cdot [0.5 \cos(2t), 0.1 \cos(2t) + \sin(2t) \cdot 0.7, -0.5 \sin(2t)]' \\
X(3) = \exp(-t) \cdot [0.5 \sin(2t), -0.7 \cos(2t) + 0.1 \sin(2t), -0.5 \cos(2t)]'
\]

Note \( v' \) is the transpose of \( v \).

What do you do if there are not \( n \) linearly independent eigenvectors? To see how to complete a fundamental set in this case look at Polking, pages 163-170.

Assignment 4:
Write down a real fundamental set if 1.
A =

-0.2  0  0.2
0.2 -0.4  0
0  0.4 -0.2

2.

A =

4  1  1  7
1  4 10  1
1 10  4  1
7  1  1  4