MATLAB 8
Numerical Methods

First consider the ode

\[ y' = f(x, y) = 2x \cdot y^2 \]
\[ y(0) = 0.1. \]

We analyze this using the Euler, Improved Euler and Runge-Kutta methods. First we make a M-file for the right hand side.

*******************************************************************************
function w=f(x,y)
w=2*x*y^2;
*******************************************************************************

Next we make M-files for each of the three methods.

*******************************************************************************
function [X,Y]=eu(x,y,xf,n)
h=(xf-x)/n;
X=x; Y=y;
for i=1:n
    y=y+h*f(x,y);
    x=x+h;
    X=[X;x];
    Y=[Y;y];
end
*******************************************************************************
function [X,Y]=imeul(x,y,xf,n)
h=(xf-x)/n;
X=x; Y=y;
for i=1:n
    k1=f(x,y);
    k2=f(x+h,y+h*k1);
    y=y+h*(k1+k2)/2;
    x=x+h;
    X=[X;x];
    Y=[Y;y];
end
*******************************************************************************
function [X,Y]=rk(x,y,xf,n)
h=(xf-x)/n;
X=x; Y=y;
for i=1:n
    k1=f(x,y);
    k2=f(x+h/2,y+h*k1/2);
    k3=f(x+h/2,y+h*k2/2);
    k4=f(x+h,y+h*k3);
    y=y+h*(k1+2*k2+2*k3+k4)/6;
    x=x+h;
    X=[X;x];
    Y=[Y;y];
end
*******************************************************************************

Here \( (x, y) \) are the initial values, \( xf \) is the final \( x \)-value and \( n \) is the number of partitions. 
\( [X, Y] \) is the \( (n+1) \times 2 \) matrix representing the computed nodes. 
To plot all three you go to the command window and enter

*******************************************************************************
\[ [z,w] = \text{eu}(0,0.1,3,20); \]
\[ [s,t] = \text{imeul}(0,0.1,3,20); \]
\[ [u,v] = \text{rk}(0,0.1,3,20); \]
plot(z,w,s,t,'--',u,v,'o')

******************************************************************************
Euler is given by a solid curve, Improved Euler by '--'s and
Runge-Kutta by 'o's.

ASSIGNMENT 8:

1. Let \( f(x,y) \) be as above. Type in the M-files for f.m and the three
approximations.
Plot their graphs for \( n=20 \) and \( x_f=3 \).

The actual solution is \( y(x) = 1/(10-x^2) \).
To find the distance between \( y(x) \) and the Euler approximation on the
interval \([0,3]\) for a given value of \( n \) type:

******************************************************************************
\[ x = 0:3/n:3; \]
\[ y = 1/(10-x.\text{^}2); \]
\[ y = y'; \]
% Here we have transposed \( y \) from a row vector to a column vector.
\[ [z,w] = \text{eu}(0,0.1,3,n); \]
\[ \text{max}(\text{abs}(y-w)) \]

******************************************************************************
Here
\[ \text{abs}(y-w) = [\text{abs}(y(1)-w(1)), \ldots, \text{abs}(y(n+1)-w(n+1))]' \]
and \( \text{max}(\text{abs}(y-w)) \) is the maximum of the \( n+1 \) components.
The theory predicts that

\[ (*) \quad \text{max}(\text{abs}(y-w)) < C*(3/n) \]
for some constant \( C \) and each \( n>1 \).

2. Set \( C_1(n) = \text{max}(\text{abs}(y-w)) / (3/n) \).
Compute \( C_1(n) \) for \( n=100, n=200, \ldots, n=800 \).
Does \( C_1(n) \) grow as \( n \) gets large or tend to level off? Is this consistent
with (*)?

Set \( C_2(n) = \text{max}(\text{abs}(y-t)) / (3/n)^2 \) for the Improved Euler method.
Compute \( C_2(n) \) for the same values of \( n \).
What should \( C_3(n) \) be for the Runge-Kutta method? Compute \( C_3(n) \).