Extra Credit Problems, Due Before Exam 1

1. Prove that for all $x$, $y$, $f$ and $g$, the following statement is a theorem of set theory:

   If $f$ and $g$ are functions, then

   $$f[x] \cap g[y] = \emptyset \iff (g^{-1} \circ f) \cap (x \times y) = \emptyset.$$ 

2. Prove that for all $x$, $y$, and $f$, the following is a theorem of set theory:

   If $f$ is a function mapping $x$ onto $y$, then there is a one-to-one function mapping $\mathcal{P}(y)$ into $\mathcal{P}(x)$. 