The Table of Integrals (pages 558-559 of the text) may be used. The Table of Integrals will be attached to the final exam.

1. If \( f(x) = x^2 - 1 \), calculate \( f\left(\frac{1}{2}\right) \) and \( \frac{1}{f(2)} \).
   - A. \( f\left(\frac{1}{2}\right) = \frac{1}{3}; \frac{1}{f(2)} = \frac{3}{4} \) B. \( f\left(\frac{1}{2}\right) = -\frac{3}{4}; f(2) = \frac{1}{3} \) C. \( f\left(\frac{1}{2}\right) = -\frac{3}{4}; f(2) = -\frac{4}{3} \) D. \( f\left(\frac{1}{2}\right) = 3; \frac{1}{f(2)} = \frac{1}{3} \) E. None of these.

2. Find the slope of the line containing the points \((-2, 4)\) and \((6, -3)\).
   - A. 4 B. \(-\frac{7}{8}\) C. \(\frac{1}{4}\) D. \(-\frac{7}{8}\) E. \(-\frac{1}{2}\)

3. Suppose 280 tons of corn were harvested in 5 days and 940 tons in 20 days. If the relationship between tons \( T \) and days \( d \) is linear, express \( T \) as a function of \( d \).
   - A. \( T = 5d + 280 \) B. \( T = -44d + 500 \) C. \( T = 44d + 60 \) D. \( T = 60d + 44 \) E. \( T = 60d + 280 \)

4. The domain of \( f(x) = \frac{1}{\sqrt{x - 1}} \) is all real numbers \( x \) such that
   - A. \( x \neq 1 \) B. \( x > 1 \) C. \( x > 0 \) D. \( x \neq 0 \) E. \(-1 < x < 1 \)

5. If \( f(x) = \sqrt{x + 1} \) and \( g(x) = x^2 + 7 \) then \( f \circ g(-1) = \)
   - A. 0 B. 3 C. \( \sqrt{7} \) D. 7 E. \( \sqrt{3} \)

6. \( \lim_{x \to 1} \frac{x^2 + 4x - 5}{x^2 - 1} = \)
   - A. \( \infty \) B. 0 C. 3 D. -3 E. 5

7. If \( f(x) = \frac{2}{x} \), find a simplified form for the difference quotient \( \frac{f(x + h) - f(x)}{h} \).
   - A. \( -\frac{2}{x^2} \) B. \( \frac{2}{x + h} - \frac{2}{x} \) C. \( \frac{2}{x(x + h)} \) D. \( -\frac{2}{x(x + h)} \) E. \( -\frac{2}{x^2 + h} \)

8. For what value of \( a \) does the function \( f(x) = x^2 + ax \) have a relative minimum at \( x = 1 \).
   - A. -2 B. 0 C. 2 D. -1 E. 1

9. The derivative of \( \frac{x^2 + 1}{x + 5} \) is
   - A. \( \frac{(x + 5)2x - (x^2 + 1)}{(x + 5)^2} \) B. \( 2x \cdot \frac{(x + 5)^2}{(x^2 + 1)^2} \cdot 2x \)
   - C. \( \frac{(x^2 + 1) + (x + 5)2x}{(x + 5)^2} \) D. \( \frac{(x^2 + 1) - (x + 5)2x}{(x + 5)^2} \) E. None of these.

10. If \( y = (3 - x^2)^3 \) then \( \frac{d^2y}{dx^2} = \)
    - A. \(-6x(3 - x^2)^2 \) B. \( 24x^2(3 - x^2) - 6(3 - x^2)^2 \) C. \( 6(3 - x^2) \)
     D. \( 24x^2(3 - x^2) \) E. None of these.

11. The line tangent to the graph of \( f(x) = x - \frac{1}{x} \) at \( x = 2 \) has slope
    - A. 5/4 B. 3/4 C. 3/2 D. 0 E. 1/4

12. A total cost function is given by \( C(x) = 1000\sqrt{x^2 + 2} \). Calculate \( C'(10) \). Give your answer correct to two decimal places.
    - A. 10,099.50 B. 990.15 C. 49.51 D. 99.01 E. 499.15

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13. Find all open intervals on which the function \( f(x) = 2x^3 - 3x^2 - 12x + 12 \) is increasing.
   A. \((-1, 2)\) B. \((-\infty, -1)\) C. \((2, \infty)\) D. \((-\infty, -1)\) and \((2, \infty)\) E. \((-1, 2)\) and \((2, \infty)\)

14. If the concentration \( C(t) \) of a certain drug remaining in the bloodstream \( t \) minutes after it is injected is given by \( C(t) = t/(5t^2 + 125) \), then the concentration is a maximum when \( t = \)
   A. 25 B. 15 C. 5 D. 10 E. There is no maximum

15. If \( f(x) = 2x^4 - 6x^2 \) then which one of the following is true?
   A. \( f \) has a relative max. at \( x = \pm \sqrt{3}/2 \) and a relative min at \( x = 0 \).
   B. \( f \) has a relative max. at \( x = 0 \) and a relative min. at \( x = \pm \sqrt{3}/2 \).
   C. \( f \) has a relative max. at \( x = -\sqrt{3}/2 \) and a relative min. at \( x = \sqrt{3}/2 \).
   D. \( f \) has no relative max. points, but has relative min. at \( x = \pm \sqrt{3}/2 \).
   E. None of these.

16. The derivative of a function \( f \) is \( f'(x) = x^2 - \frac{8}{x} \). Then at \( x = 2 \), \( f \) has
   A. an inflection point  B. a relative maximum  C. a vertical tangent  D. a vertical asymptote  E. a relative minimum

17. If \( f(x) = \frac{1}{3}x^3 - 9x + 2 \). Then on the closed interval \( 0 \leq x \leq 4 \),
   A. \( f \) has an absolute max. at \( x = 3 \) and an absolute min. at \( x = 0 \).
   B. \( f \) has an absolute max. at \( x = 4 \) and an absolute min. at \( x = 3 \).
   C. \( f \) has an absolute max. at \( x = 0 \) and an absolute min. at \( x = 4 \).
   D. \( f \) has an absolute max. at \( x = 0 \) and an absolute min. at \( x = 3 \).
   E. None of these.

18. A total-cost function is given by \( C(x) = 1000\sqrt{x^3 + 1} \). Find the marginal cost when \( x = 2 \).
   A. $166.67  B. $333.33  C. $4000  D. $2000  E. $1000

19. A display case is in the shape of a rectangular box with a square base and open top. Suppose the volume is 21 cubic ft If \( x \) is the length of one side of the base, what value should \( x \) have to minimize the surface area? Give your answer correct to two decimal places.

20. A manufacturer determines that in order to sell \( x \) units of a product, the price per unit must be \( p = 1000 - x \). The manufacturer also determines that the total cost of producing \( x \) units is \( C(x) = 3000 + 20x \). Calculate the maximum profit.
   A. $490  B. $237,100  C. $121,500  D. $23,000  E. There is no maximum.

21. If \( y = e^{x^2} \) then \( \frac{dy}{dx} = \)
   A. \( e^{x^2} \)  B. \( x^2e^{x^2-1} \)  C. \( 2xe^{x^2} \)  D. \( 2xe^{x^2} \)  E. \( e^{2x} \)

22. If \( y = \ln(1 - x^2) \) then \( \frac{dy}{dx} = \)
   A. \( \frac{1}{1 - x^2} \)  B. \( \frac{2x}{\sqrt{1 - x^2}} \)  C. \( \frac{-2x}{1 - x^2} \)  D. \( \frac{1}{2(1 - x^2)} \)  E. \( \frac{2x}{1 - x^2} \)

23. A population grows exponentially \( (Q = Q_0e^{kt}) \). In 1960 it was 50,000 and in 1965 it was 100,000. What was the population in 1970?
   A. 200,000  B. 150,000  C. 250,000  D. 300,000  E. 225,000
24. Evaluate \( \int_{0}^{1} 3\sqrt{2x + 1} \, dx \).
   A. 27  B. 48  C. 52  D. 26  E. 35

25. Evaluate \( \int \left( \frac{2}{x} - \sqrt{x} \right) \, dx \).
   A. \( \ln x - \frac{2}{\sqrt{x} + x} + C \)  B. \( -\frac{2}{x^2} - \frac{1}{2}x^{-1/2} + C \)  C. \( 2\ln x - \frac{2x^{3/2}}{3} + C \)  
   D. \( -2/x^2 - 2x^{3/2}/3 + C \)  E. \( 2\ln x - 1/2\sqrt{x} + C \)

26. Evaluate \( \int (3x - 1)^{-4} \, dx \).
   A. \( -(12)(3x - 1)^{-5} + C \)  B. \( -\frac{1}{9}(3x - 1)^{-3} + C \)  C. \( (3x - 1)^{-3} + C \)
   D. \( -\frac{1}{3}(3x - 1)^{-3} + C \)  E. \( -12(3x - 1)^{-5} + C \)

27. Evaluate \( \int e^{3-2x} \, dx \).
   A. \( -2e^{3-2x} + C \)  B. \( -\frac{1}{2}e^{3-2x} + C \)  C. \( \frac{e^{4-2x}}{4-2x} \)  D. \( \frac{1}{3}e^{3-2x} + C \)  E. \( \frac{e^{3-2x}}{3-2x} + C \)

28. Evaluate \( \int_{1}^{2} \frac{dx}{3x + 1} \). Give your answer correct to four decimal places.
   A. 0.5596  B. 0.6486  C. 1.9459  D. 0.0810  E. 0.1865

29. Evaluate \( \int_{0}^{1} x(x^2 + 1)^5 \, dx \).
   A. \( \frac{21}{4} \)  B. \( \frac{16}{3} \)  C. \( \frac{21}{2} \)  D. \( \frac{32}{3} \)  E. \( \frac{24}{3} \)

30. The area of the region bounded by the curves \( y = x^2 + 1 \) and \( y = 3x + 5 \) is
   A. \( \frac{125}{6} \)  B. \( \frac{56}{3} \)  C. \( \frac{27}{2} \)  D. \( \frac{25}{6} \)  E. \( \frac{32}{3} \)

31. Use the table of integrals to evaluate \( \int 2x \ln x \, dx \).
   A. \( x^2 \ln x - \frac{x^2}{2} + C \)  B. \( \frac{1}{2}x^2 \ln x - \frac{1}{2}x^2 + C \)  C. \( \frac{1}{2}x^2 \ln x - \frac{1}{6}x^3 + C \)
   D. \( x \ln x^2 + 1/x + C \)  E. None of these.

32. Use the table of integrals to evaluate \( \int \frac{dx}{\sqrt{x^2 - 9}} \).
   A. \( \frac{1}{6} \ln \left| \frac{x-3}{x+3} \right| + C \)  B. \( \ln |x + \sqrt{x^2 - 9}| + C \)  C. \( \ln |x - \sqrt{x^2 - 9}| + C \)
   D. \( \ln |x + \sqrt{x^2 + 9}| + C \)  E. None of these.

33. Find the average value of \( f(x) = x^2 \) over the interval \( 1 \leq x \leq 4 \).
   A. \( \frac{17}{2} \)  B. \( \frac{15}{2} \)  C. 21  D. \( \frac{65}{3} \)  E. 7

34. Find a function \( f \) whose tangent line has slope \( x\sqrt{5 - x^2} \) for each value of \( x \) and whose graph passes through the point \( (2,10) \). \( f(x) = \)
   A. \( -\frac{1}{2}(5 - x^2)^{3/2} \)  B. \( \frac{2}{3}(5 - x^2)^{3/2} + \frac{28}{3} \)  C. \( \frac{1}{2}(5 - x^2)^{3/2} + \frac{20}{3} \)
   D. \( -\frac{1}{3}(5 - x^2)^{3/2} + \frac{31}{3} \)  E. \( \frac{2}{3}(5 - x^2)^{3/2} + \frac{17}{2} \)

35. It is estimated that \( t \) years from now the population of a certain town will be increasing at a rate of \( 5 + 3t^{2/3} \) hundred people per year. If the population is presently 100,000, by how many people will be population increase over the next 8 years?
   A. 100  B. 9,760  C. 6,260  D. 24,760  E. 17,260
36. Calculate, if possible, the following improper integral \( \int_0^\infty xe^{-x^2} \, dx \)

A. \(-\frac{1}{2}\)  B. 1  C. \(\frac{1}{2}\)  D. \(\frac{5}{2}\)  E. The integral diverges.

37. Find the value of \(k\) so that \(f(x) = k(3 - x)\) is a probability density function on the interval \([0,3]\).

A. \(k = \frac{1}{9}\)  B. \(k = -\frac{2}{3}\)  C. \(k = -\frac{1}{3}\)  D. \(k = \frac{2}{9}\)  E. \(k = \frac{1}{6}\)

38. Records indicate that \(t\) hours past midnight, the temperature at the West Lafayette airport was \(f(t) = -0.3t^2 + 4t + 10\) degrees Celsius. What was the average temperature at the airport between 2:00 A.M. and 7:00 A.M.? (Give your answer to the nearest degree.)

A. 3°  B. 27°  C. 21°  D. 5°  E. 18°

39. Let \(f(x)\) be the probability density function defined on the interval \([0,\infty]\) by \(f(x) = \frac{3}{x^4}\).

Calculate \(P(2 \leq x < \infty)\).

A. 1  B. \(\frac{3}{8}\)  C. \(\frac{1}{2}\)  D. \(\frac{1}{7}\)  E. \(\frac{1}{8}\)

40. The probability density function for the life span of light bulbs manufactured by a certain company is \(f(t) = 0.01e^{-0.01t}\) where \(t\) denotes the life span in hours of a randomly selected bulb, \(0 \leq t < \infty\). What is the probability that the life span of a randomly selected bulb is less than or equal to 10 hours? Give your answer correct to three decimal places.

A. 0.009  B. 0.095  C. 0.905  D. 0.090  E. 0.303

Answers