1. You must use a #2 pencil on the mark-sense sheet (answer sheet).

2. If the cover of your question booklet is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.

3. On the mark-sense sheet, fill in your TA’s name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.


7. Fill in your name and your instructor’s name on the question sheets above.

8. There are 12 questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work on the question sheets.

9. Turn in both the mark-sense sheets and the question sheets when you are finished.

10. If you finish the exam before 8:50, you may leave the room after turning in the scantron sheet and the exam booklet. If you don’t finish before 8:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.
EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else’s test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME:  

STUDENT SIGNATURE:  
1. Find the integral $\int_{0}^{\frac{\pi}{4}} \tan^5 x \sec x \, dx$

A. $\frac{1}{15}(3\sqrt{2} - 4)$
B. $\frac{1}{15}(4\sqrt{2} - 5)$
C. $\frac{1}{15}(5\sqrt{2} - 6)$
D. $\frac{1}{15}(6\sqrt{2} - 7)$
E. $\frac{1}{15}(7\sqrt{2} - 8)$

2. Find the integral $\int_{0}^{\frac{\pi}{4}} \cos^4 x \, dx$. You may need to use the half-angle identities

$\cos^2 x = \frac{1 + \cos 2x}{2}$ and/or $\sin^2 x = \frac{1 - \cos 2x}{2}$

A. $\frac{1}{32}(\pi + 5)$
B. $\frac{1}{32}(2\pi + 7)$
C. $\frac{1}{32}(3\pi + 8)$
D. $\frac{1}{32}(2\pi + 5)$
E. $\frac{1}{32}(\pi + 8)$
3. What does the integral \( \int x^4 \sqrt{1 + x^2} \, dx \) become after a trigonometric substitution?

A. \( \int \sec^4 \theta \tan^3 \theta \, d\theta \)
B. \( \int \sec^3 \theta \tan^4 \theta \, d\theta \)
C. \( \int \sec^3 \theta \tan^3 \theta \, d\theta \)
D. \( \int \sec^3 \theta \tan^2 \theta \, d\theta \)
E. \( \int \sec^2 \theta \tan^3 \theta \, d\theta \)

4. Compute \( \int_{0}^{4} \sqrt{16 - x^2} \, dx \)

A. \( \frac{\pi}{2} \)
B. \( \pi \)
C. \( 2\pi \)
D. \( 4\pi \)
E. \( 6\pi \)
5. Which of the following is the partial fraction expansion of \( \frac{4x^3 - x}{x^2(x^2 + 1)^2} \)?

A. \( \frac{A}{x^2} + \frac{B}{x^2 + 1} + \frac{C}{(x^2 + 1)^2} \)

B. \( \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^2 + 1} + \frac{D}{(x^2 + 1)^2} \)

C. \( \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2} \)

D. \( \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^2 + 1} + \frac{E}{(x^2 + 1)^2} \)

E. \( \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^2 + 1} + \frac{D}{x^2 + 1} + \frac{E}{(x^2 + 1)^2} \)

6. Compute \( \int \frac{3x + 1}{x^2 + x} \, dx \)

A. \( 6 \ln |x| + 5 \ln |x + 1| + C \)

B. \( 3 \ln(x^2) + \ln |x| + C \)

C. \( 3 \ln |x^2 + x| + C \)

D. \( \ln |x| - \ln |x + 1| + C \)

E. \( \ln |x| + 2 \ln |x + 1| + C \)
7. For the function $f(x)$ whose values are given in the table below, approximate $\int_0^4 f(x) \, dx$ using the Midpoint Rule with 2 subintervals.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0</td>
<td>1.2</td>
<td>4.3</td>
<td>6.5</td>
<td>1</td>
</tr>
</tbody>
</table>

A. 15.4  
B. 12.5  
C. 12  
D. 16.7  
E. 13

8. Use the integral formula

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln |u + \sqrt{u^2 - a^2}| + C$$

to find $\int \frac{dx}{\sqrt{8x^2 - 5}} \, dx$.

A. $\frac{1}{\sqrt{8}} \ln |x + \sqrt{8x^2 - 5}| + C$  
B. $\frac{1}{\sqrt{8}} \ln |x + \sqrt{x^2 - \frac{5}{8}}| + C$  
C. $\sqrt{8} \ln |x + \sqrt{x^2 - \frac{5}{8}}| + C$  
D. $\ln |x + \sqrt{x^2 - \frac{5}{8}}| + C$  
E. $\ln |\frac{x}{\sqrt{8}} + \frac{1}{\sqrt{8}} \sqrt{x^2 - \frac{5}{8}}| + C$
9. Which of the following improper integrals are convergent?

I. \[ \int_{-\infty}^{0} \frac{1}{2 - 8x} \, dx \]

II. \[ \int_{1}^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} \, dx \]

III. \[ \int_{-1}^{2} \frac{1}{x^2} \, dx \]

A. I, II, III  
B. II only  
C. III only  
D. I, II only  
E. II, III only

10. The derivative of the function \( g(x) \) is \( g'(x) = \sqrt{\sec^2 x \tan^2 x - 1} \). What is the length of the curve \( y = g(x) \) on the interval \( 0 \leq x \leq \frac{\pi}{4} \)?

A. \( \sqrt{2} - 1 \)  
B. \( \frac{\sqrt{2}}{2} - 1 \)  
C. \( \frac{\sqrt{2}}{2} \)  
D. \( \sqrt{2} \)  
E. 1
11. If the centroid of the region bounded by \( y = x^2 \) and \( y = x \) is \((\bar{x}, \bar{y})\). Find \( \bar{y} \).

A. \( \frac{1}{2} \)
B. \( \frac{1}{6} \)
C. \( \frac{2}{5} \)
D. \( \frac{1}{12} \)
E. \( \frac{1}{15} \)

12. Which of the following statements is/are true about the sequence \( a_n = \frac{5n - 7}{7n + 3} \)?

I. The sequence is convergent
II. The sequence is bounded
III. The sequence is not monotonic

A. I only
B. II only
C. III only
D. I and II only
E. I and III only