1. An observer is stationed 300 feet from a rocket launch pad. The rocket rises vertically off the launch pad. A few seconds after takeoff, the rocket is 300 feet in the air and rising at 100 feet/sec. How fast is the angle of elevation, $\theta$, changing at that instant?

\[
\theta = \tan^{-1} \left( \frac{h}{300} \right)
\]

\[
\frac{d\theta}{dt} = \frac{1}{1 + \left( \frac{h}{300} \right)^2} \left( \frac{1}{300} \right) \frac{dh}{dt}
\]

AT INSTANT:

\[
\frac{1}{1 + \left( \frac{300}{300} \right)^2} \left( \frac{1}{300} \right) (100)
\]

A. $\frac{1}{6}$ radian/sec
B. 2 radian/sec
C. $\frac{1}{2}$ radian/sec
D. $\frac{1}{4}$ radian/sec
E. $\frac{2}{3}$ radian/sec

2. Use a linear approximation (or differentials) to estimate the value of $\sqrt{24.8}$

\[
\sqrt{25} + \frac{1}{2\sqrt{25}} (24.8 - 25)
\]

A. 5.20
B. $\boxed{4.98}$
C. 4.95
D. 4.92
E. 4.90
3. Find the minimum value of $f(x) = x^3 - x$ on the closed interval $[-1, 1]$.

Hint: Find the actual value of $f$ and NOT the $x$-value at which that minimum occurs.

A. 0
B. $-\frac{1}{\sqrt{3}}$
C. $-\frac{1}{3}$
D. $-\frac{2}{3\sqrt{3}}$

E. There is no absolute minimum value.

$f'(x) = 3x^2 - 1$

Crit numbers: $x = \pm\frac{1}{\sqrt{3}}$

$f(-1) = 0$
$f(-\sqrt{3}) = \frac{2}{3\sqrt{3}}$
$f(\sqrt{3}) = -\frac{2}{3\sqrt{3}} $
$f(1) = 0$

4. The function $f$ is continuous on $[0, 2]$ and differentiable on $(0, 2)$, and consequently the direct application of the mean value theorem guarantees the existence of $c$, where $c$ is between 0 and 2 and pictured below. Find $f'(c)$.

\[
\frac{f(2) - f(0)}{2 - 0} = \frac{1 - 5}{2} = -2
\]

A. $-2$
B. 1.2
C. 0.5
D. $-4$
E. $-1$
5. Which statement accurately describes the function

\[ f(x) = x^4 - 6x^3 \]
on the interval \((0, 3)\)?

A. \( f \) is increasing and its graph is concave up.
B. \( f \) is decreasing and its graph is concave up.
C. \( f \) is increasing and its graph is concave down.
D. \( f \) is decreasing and its graph is concave down.
E. None of the above.

\[ f'(x) = 4x^3 - 18x^2 \]
\[ = 2x^2(2x - 9) \]
\[ f''(x) = 12x^2 - 36x \]
\[ = 12x(x - 3) \]

6. The graph of \( y = f'(x) \), the derivative of \( f \), is shown below.

Which of the following statements about \( f \) are true?

I. The graph of \( f \) is concave up on the interval \((2, 4)\).
II. \( f(x) \) has a local minimum at \( x = 2 \).
III. \( (1, f(1)) \) is an inflection point for \( f \).

A. None of these statements are true.
B. I and III
C. II and III
D. I and II
E. I, II, and III
7. Find the limit.

\[
\lim_{x \to 0} \frac{\tan x - x}{x^3} = \lim_{x \to 0} \frac{\sec^2 x - 1}{3x^2} = \lim_{x \to 0} \frac{2\sec x (\sec x \tan x)}{6x} = \lim_{x \to 0} \frac{\tan x}{3\cos^2 x} = \frac{1}{3+0} = \frac{1}{3}
\]

A. \( -\infty \)
B. \( \frac{1}{3} \)
C. 1
D. \( -\frac{1}{6} \)
E. 0

8. Find the \( x \)-coordinate of the inflection point of the function \( f(x) = \frac{1}{\ln x} \) on the interval \( 0 < x < 1 \).

\[
f'(x) = -\left(\ln x\right)^2 \left(\frac{1}{x}\right) = \frac{-1}{(\ln x)^2} x
\]

\[
f''(x) = \frac{2(\ln x)(\frac{1}{x})x + (\ln x)^2}{(\ln x)^4} x^2
\]

\[
2 \ln x + (\ln x)^2 = 0
\]

\[
\ln x \left(2 + \ln x\right) = 0
\]

\[
\ln x = -2
\]

\[
x = e^{-2}
\]
9. A six-sided box is to have four clear plastic sides, a wooden square top, and a wooden square bottom. The volume of the box must be 24 ft³. Plastic costs $1 per ft² and wood costs $3 per ft². Find the dimensions of the box which minimize cost.

\[ \text{Volume} = x^2 y = 24 \Rightarrow y = \frac{24}{x^2} \]

\[ \text{MIN} \]

\[ \text{cost} = 1(4xy) + 3(2x^2) = 4x \left( \frac{24}{x^2} \right) + 6x^2 \]

\[ C(x) = \frac{96}{x} + 6x^2 \]

\[ C'(x) = -\frac{96}{x^2} + 12x = \frac{-96 + 12x^3}{x^2} \]

\[-96 + 12x^3 = 0 \Rightarrow x^3 = 8 \Rightarrow x = 2 \]

10. A rectangle is formed with one corner at (0,0) and the opposite corner on the graph of \( y = -\ln x \), where 0 < x < 1. What is the largest possible area of such a rectangle?

\[ \text{MAX} \]

\[ \text{Area} = xy \]

\[ A(x) = x \left( -\ln x \right) \]

\[ A'(x) = -\ln x - x \left( \frac{1}{x} \right) \]

\[-\ln x - 1 = 0 \]

\[ \ln x = -1 \]

\[ x = \frac{1}{e} \]

\[ y = -\ln(e^{\frac{1}{e}}) = \frac{1}{e} \]

\[ xy = \left( e^{\frac{1}{e}} \right) \left( \frac{1}{e} \right) \]

A. \( \frac{\sqrt{e}}{2} \)

B. \( e \)

C. \( \frac{1}{e} \)

D. \( \frac{\ln 2}{2} \)

E. There is no maximum.
11. Suppose \( f \) is a differentiable function with \( f''(x) > 0 \) for all real numbers \( x \). Assume that \( f(1) = 3 \) and \( f(5) = 3 \). Which one of these statements must be true?

A. \( f'(x) \) is decreasing at \( x = 3 \).
B. \( f(x) \geq 0 \) for all real numbers \( x \).
C. \( f \) has an inflection point.
D. \( f'(3) > 0 \).
E. \( f \) has a local minimum.

**ROLLES:**
\[ f'(c) = 0 \] for some \( 1 < c < 5 \)

**SECOND DERIVATIVE TEST:**
\[ f''(c) > 0 \Rightarrow \text{LOCAL MIN} \] at \( c \)

12. Which of these curves is the graph of \( y = 5x^6 + 6x^5 \)?

- **A.**
  \[ y' = 30x^4(x+1) \]
  \[ y'' = 30x^3(5x+4) \]

- **B.**
  \[ y' \] is DECR: \( (-\infty,-1) \) \quad \text{INCR:} \( (-1,0) U (0, \infty) \)

- **C.**
  \[ y' \] is CONCAVE DOWN: \( (-\frac{4}{5},0) \) \quad \text{UP:} \( (-\infty,-\frac{4}{5}) U (0, \infty) \)

- **D.**

- **E.**