PROBLEM 1: The angle between the planes given by the equations

\[ x + y = 2 \text{ and } x + y + \sqrt{2}z = \sqrt{6} \]

is

A. \( \frac{\pi}{2} \)
B. \( \frac{\pi}{4} \)
C. \( \frac{\pi}{6} \)
D. \( \pi \)
E. \( \frac{\pi}{3} \)

\[ \vec{n}_1 = \langle 1, 1, 0 \rangle, \quad \vec{n}_2 = \langle 1, 1, \sqrt{2} \rangle \]

\[ \cos \theta = \vec{n}_1 \cdot \vec{n}_2 = \frac{1+1+0}{1 \cdot 1 \cdot 1 \cdot 1} = \frac{1}{\sqrt{2} \cdot \sqrt{4}} \]

\[ \Rightarrow \theta = \frac{\pi}{4} \]

PROBLEM 2: a) Consider the following two curves \( r_1(t) = (t, t^2, t^3) \) and \( r_2(t) = (-1 + 3t, 1 + 3t, -1 + 9t) \). The curves have

A. 2 intersection points and no points of collision.
B. 1 intersection point, which is a point of collision.
C. 2 intersection points, one of which is a point of collision.
D. 1 intersection point and no points of collision.
E. 2 intersection points which both are points of collision.

Solve \( \vec{r}_1(s) = \vec{r}_2(t) \),

ie \( s = -1 + 3t, \quad s^2 = 1 + 3t, \quad s^3 = -1 + 9t \)

\[
\begin{align*}
1 & \quad s = -1 + 3t \\
2 & \quad s^2 = 1 + 3t \\
3 & \quad s^3 = -1 + 9t \\
\end{align*}
\]

(1) \& (2) \Rightarrow 1 + 3t = s^2 = (3t - 1)^2

\[ = 9t^2 - 6t + 1 \]

\[ \Rightarrow 0 = 9t^2 - 6t + 1 \]

\[ \Rightarrow t = 0 \text{ or } t = 1 \]

\[ t = 0 \Rightarrow s = -1 \Rightarrow \text{intersection } @ (-1, 1, -1) \]

\[ t = 1 \Rightarrow s = 2 \Rightarrow \text{intersection } @ (3, 4, 8) \]
PROBLEM 3: Find the length of the curve given by
\[ \mathbf{r}(t) = (2t, 4\sqrt{t}, \ln t) \]
for \(1 \leq t \leq e\).

A. \( e - 1 \)
B. \( 2e + 1 \)
C. \( e + 1 \)
D. \( 2e - 1 \)
E. \( 4e - 3 \)

\[ \mathbf{r}'(t) = <2, 2t^{-1/2} \frac{1}{t}, \frac{1}{t}> \]
\[ |\mathbf{r}'(t)| = \sqrt{4 + \frac{4}{t^2} + \frac{1}{t^2}} = \sqrt{\frac{4t^2 + 4t + 1}{t^2}} = \frac{2t + 1}{t^2} \]

\[ L = \int_1^e \left( 2 + \frac{1}{t^2} \right) dt = \left[ 2t + \ln t \right]_1^e \]
\[ = (2e + 1) - 0 \]
\[ = 2e - 1 \]

PROBLEM 4: A particle is moving with acceleration \( \vec{a} = t \vec{j} + \vec{k} \). If the velocity at time \( t = 1 \) is \( \vec{v}(1) = \vec{i} - \frac{1}{2} \vec{j} \), what is the velocity at time \( t = 0 \)?

A. \( \vec{i} - \vec{j} - \vec{k} \)
B. \( \vec{i} - \vec{j} + 2\vec{k} \)
C. \( \vec{i} + \frac{1}{2} \vec{j} - \vec{k} \)
D. \( \vec{i} + \frac{1}{2} \vec{j} - \vec{k} \)
E. \( -\vec{i} + \vec{j} \)

\[ \vec{a} = \vec{v}' = t \vec{j} + \vec{k}, \quad \vec{v}(t) = \vec{i} - \frac{1}{2} \vec{j} \]
\[ \Rightarrow \vec{v}'(t) = \frac{t}{2} \vec{j} + t \vec{k} + \vec{c} \]

and \( \vec{v}'(1) = \frac{1}{2} \vec{j} + k + \vec{c} = \vec{i} - \frac{1}{2} \vec{j} \)

\[ \Rightarrow \vec{c} = \vec{i} - \vec{j} - \vec{k} \]
\[ \Rightarrow \vec{v}(0) = \frac{1}{2} \vec{j} + t \vec{k} + \vec{c} \bigg|_{t=0} = \vec{c} = \vec{i} - \vec{j} - \vec{k} \]
Name:

**Problem 5:** The function
\[ f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases} \]

A. is continuous on \( \mathbb{R}^2 \).
B. is not well defined at \((0, 0)\), since it evaluates to \( \frac{0}{0} \).
C. has limit 0 at \((0, 0)\) along the diagonal \( x = y \).
D. has limit 2 at \((1, 1)\).
E. is continuous on \( \mathbb{R}^2 \setminus (0, 0) \).

Along the diagonal \( y = x \), the expression \( \frac{2x^2}{x^2 + x^2} \) has limit 1, which does not equal \( f(0, 0) \). Hence, \( f \) is not continuous at \((0, 0)\). Rational functions are continuous on their domain.

**Problem 6:** Consider the function
\[ f(x, y) = 3x^2 - 2y^2 - 2x + 3xy \]

At the point \((1, 1, 2)\).
A. The slope of the tangent line to the curve of intersection of the graph of \( f \) and \( x = 1 \) is positive.
B. The gradient of \( f \) is \((-2, -1, 1)\).
C. The slope of the tangent line to the curve of intersection of the graph of \( f \) and \( y = 1 \) is positive.
D. The partial derivatives vanish.
E. The tangent plane is \( z - 2 = 5(x - 1) + 6(y - 2) \).

\[ \frac{\text{true because the slope of the tangent line to the curve of intersection of the graph of } f \text{ and } y = 1 \text{ at } (1, 1, 2)}{f_x (1, 1)} = 6x - 2 + 3y |_{(1, 1)} = 6 - 2 + 3 = 7 > 0 \]
PROBLEM 7: Consider the function 
\[ f(x, y, z) = xyz \]
which of the following is true.

1. \( df = xdx + ydy + zdz \)
2. Its linear approximation is the tangent plane.
3. If \( \Delta x = \Delta y = \Delta z = 0.2 \) then the error estimated by using differentials at \((1, 2, 1)\) is 1
4. Its gradient is \( \langle yz, xz, xy \rangle \)
5. Its linear approximation at \((1, 1, 1)\) is \( L(x, y, z) = x + y + z - 2 \).

A. 1, 2, 3 are true.
B. 3, 4, 5 are true.
C. All are true.
D. None is true.
E. Only 4 is true.

\[ \frac{df}{dx} = f_x dx + f_y dy + f_z dz \]
\[ \frac{dy}{dx} = \frac{f_y}{f_x} \]
\[ \frac{dz}{dx} = \frac{f_z}{f_x} \]

PROBLEM 8: Consider the function 
\[ g(s, t) = f(t \sin \left( \frac{\pi}{2} s \right), st^2) \]
with \( f \) differentiable. Use the table of values to calculate \( g_t(1, 2) \)

\[
\begin{array}{c|ccc}
& f & g_x & g_y \\
\hline
(1, 2) & \pi & 5 & 3 \\
(2, 4) & 5 & 2 & \end{array}
\]

A. \( 3 + 8 \pi \)
B. \( 15 + 2 \pi^2 \)
C. \( 9 \)
D. \( 18 \)
E. \( 2 \pi + 20 \)

\[
\frac{\mathrm{d}x}{\mathrm{d}t} = t \sin \left( \frac{\pi}{2} s \right) \quad \frac{\mathrm{d}y}{\mathrm{d}t} = st^2.
\]

A. \( x = t \sin \left( \frac{\pi}{2} s \right) \)

When \( s = 1, t = 2 \):

\[
x = t \sin \left( \frac{\pi}{2} s \right) = 2 \cdot 1 = 2
\]

and \( y = st^2 = 1.4 = 2 \).

So \( g_t(1, 2) = f_x(2, 4) \cdot \sin \left( \frac{\pi}{2} s \right)(1, 2) + f_y(2, 4) \cdot 2t(1, 2) \)

\[
= 2 \cdot 1 + 4 \cdot 4 = 18
\]
Problem 9: Find the maximum rate of change of the function
\[ f(x, y, z) = x^2y - 2yz + z^2x \]
at the point \((1, 1, -1)\).
A. \(\sqrt{52}\)
B. \(\sqrt{44}\)
C. \(2\sqrt{10}\)
D. \(\sqrt{34}\)
E. \(2\sqrt{34}\)

\[ \nabla f = \frac{\partial f}{\partial x} \hat{i} + \frac{\partial f}{\partial y} \hat{j} + \frac{\partial f}{\partial z} \hat{k} \]
\[ = \langle 2xy + z^2, x^2 - 2z, -2y + 2xz \rangle \]
\[ \text{at} \ (1, 1, -1) \]
\[ = \langle 3, 3, -4 \rangle \]
\[ |\nabla f| = \sqrt{9 + 9 + 16} = \sqrt{34} \]

Problem 10: Find all critical points of the function
\[ f(x, y) = x^3 + 2xy - 2y^2 - 10x \]
and classify them.
A. \((2, -1), \left(\frac{2}{3}, \frac{5}{6}\right)\) one min, one max.
B. \((-1, -2), \left(\frac{2}{3}, \frac{5}{6}\right)\) one min, one saddle.
C. \((-2, -1), \left(\frac{2}{3}, \frac{5}{6}\right)\) one min, one saddle.
D. \((-1, -2), \left(\frac{2}{3}, \frac{5}{6}\right)\) one max, one saddle.
E. \((-2, -1), \left(\frac{2}{3}, \frac{5}{6}\right)\) one max, one saddle.

\[ \begin{align*}
0 &= f_x = 3x^2 + 2y - 10 \\
0 &= f_y = 2x - 4y \\
\Rightarrow 2x &= 2y \\
\Rightarrow x &= y \\
\Rightarrow 2y^2 + 2y - 10 &= 0 \\
\Rightarrow 6y^2 + y - 5 &= 0, \\
\text{So } y &= \frac{-1 \pm \sqrt{1 + 120}}{12} = \frac{-1 \pm 11}{12} = -1 \text{ or } \frac{5}{6} \\
\implies \text{Critical pts } (-2,-1) \text{ and } \left(\frac{2}{3}, \frac{5}{6}\right) \\
\end{align*} \]

\[ f_{xx} = 6x, \quad f_{xy} = 2, \quad f_{yy} = -4 \]
\[ \Rightarrow D = f_{xx}f_{yy} - (f_{xy})^2 = 24x - 4 \]
\[ D (-2,-1) = -48 - 4 > 0. \text{ Also } f_{xx} (-2,-1) = -12 < 0. \]
\[ \implies \text{local max at } D (-2,-1). \text{ So, } \]
\[ D \left(\frac{2}{3}, \frac{5}{6}\right) = (-24)\left(\frac{5}{6}\right) - 4 = -40 - 4 < 0 \]
\[ \implies \text{saddle pt at } \left(\frac{2}{3}, \frac{5}{6}\right) \]