1. Find the inverse Laplace transform of \( \frac{s^2 + 6}{(s - 2)(s^2 + 2s + 2)} \).

2. Find the inverse Laplace transform of \( \frac{e^{-3s}}{s^2 + 4s + 5} \).

3. Compute \( u(t - 1) \ast (e^{-2t}u(t)) \) and its Laplace transform.

4. Solve \( y(t) = 2t - 4 \int_0^t y(\tau)(t - \tau)d\tau \).

5. Solve \( y'' + 2y' - 3y = 8e^{-t} + \delta(t - 1/2) \ y(0) = 3 \ y'(0) = -5 \).

6. Find the Laplace transform of the \( \pi \)-periodic extension of the function \( f(x) = x \) defined on the interval \([0, \pi]\).

7. Find all eigenfunctions and eigenvalues of the following Sturm-Liouville problem:
\[ y'' + 2y' + (2 + \lambda)y = 0, \quad y(0) = y(\pi) = 0. \]

8. Find sine, cosine and complex Fourier transforms of the function
\[ f(x) = \begin{cases} 0, & x < 0 \\ xe^{-x}, & 0 \leq x \end{cases} \]
Find also the Laplace transform of \( f \). Try not to repeat calculations.

9. Let \( f(x) = x \) be defined on the interval \([0, \pi]\).
   (1) Find the Fourier series of its \( \pi \)-periodic extension.
   (2) Find the sine and cosine series of \( f \).
   (3) Draw the graphs of sums of all three series.
   (4) Substitute \( x = 0 \) into all three series. What are the sums of the resulting series? The same question for \( x = -2 \).

10. Suppose the Fourier series of some \( 2\pi \)-periodic function \( f(x) \) starts from \( 1 + \sin x + \cos x \) (i.e. \( a_0 = a_1 = b_1 = 1 \)). Could it happen that \( |f(x)| \leq 1 \) for all \( x \)? (Hint: use Parseval’s equality/Bessel’s inequality).