MA 174: Multivariable Calculus

EXAM II (practice)

NAME ______________________  INSTRUCTOR ______________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (5 pts) ________  9. (5 pts) ________
2. (5 pts) ________  9. (5 pts) ________
3. (5 pts) ________  9. (5 pts) ________
4. (5 pts) ________  9. (5 pts) ________
5. (5 pts) ________  9. (5 pts) ________
6. (5 pts) ________  9. (5 pts) ________

Total Points: ___________
1. Suppose \( z = f(x,y) \), where \( x = e^t \) and \( y = t^2 + 3t + 2 \). Given that \( \frac{\partial z}{\partial x} = 2xy^2 - y \) and \( \frac{\partial z}{\partial y} = 2x^2y - x \), find \( \frac{dz}{dt} \) when \( t = 0 \).

A. 3  
B. 6  
C. 15  
D. 9  
E. -1

2. Find the directional derivative of the function \( f(x,y,z) = x^2y^2z^6 \) at the point \((1,1,1)\) in the direction of the vector \((2,1,-2)\).

A. -6  
B. -2  
C. 0  
D. 2  
E. 6

3. Find the direction in which the function \( z = x^2 + 3xy - \frac{1}{2} y^2 \) is increasing most rapidly at \((-1,-1)\).

A. 3i  
B. 5i + 2j - k  
C. -5i - 2j  
D. 2i - 5j  
E. \( \sqrt{29} \)
4. If \( xz^3 - xyz = 4 \), find \( \frac{\partial z}{\partial x} \).

A. \( \frac{\partial z}{\partial x} = \frac{xz}{z^3 - y^2} \)
B. \( \frac{\partial z}{\partial x} = \frac{3xz^2 - xy}{z^3 - yz} \)
C. \( \frac{\partial z}{\partial x} = 2x + xy \)
D. \( \frac{\partial z}{\partial x} = \frac{yz - z^3}{3xz^2 - xy} \)
E. \( \frac{\partial z}{\partial x} = z^3 - yz \)

5. The directional derivative of \( f(x, y) = x^3e^{-2y} \) in the direction of greatest increase of \( f \) at the point \( (1, 0) \) is

A. 6
B. 5
C. \( \sqrt{5} \)
D. 13
E. \( \sqrt{13} \)

6. By using a linear approximation of \( f(x, y) = \sqrt{x^2 + y} \) at \( (4, 9) \), compute the approximate value of \( f(5,8) \).

A. 5.2
B. 5.3
C. 5.5
D. 5.7
E. 5.9
7. The max and min values of \( f(x, y, z) = xyz \) on the surface \( 2x^2 + 2y^2 + z^2 = 2 \) are

A. \( \pm \frac{\sqrt{2}}{9} \)
B. \( \pm \frac{\sqrt{3}}{9} \)
C. \( \pm \frac{\sqrt{6}}{9} \)
D. \( \pm \frac{2\sqrt{2}}{9} \)
E. \( \pm \frac{2\sqrt{3}}{9} \)

8. Find the maximum value of \( x^2 + y^2 \) subject to the constraint \( x^2 - 2x + y^2 - 4y = 0 \).

A. 0
B. 2
C. 4
D. 16
E. 20

9. If we use the method of Lagrange multipliers to find the maximum of \( f(x, y) = 2x^2 - y^2 - y \) subject to the constraint \( x^2 + y^2 = 1 \), the Lagrange multipliers \( \lambda \) that we find are:

A. \( \lambda = 2 \)
B. \( \lambda = 0 \)
C. \( \lambda = -1 \)
D. \( \lambda = 2 \) and \( \lambda = -1 \)
E. \( \lambda = 0 \) and \( \lambda = -1 \)
10. For the function \( f(x, y) = x^3 + 2y^2 + xy - 2x + 5y \), the point \((-1, -1)\) yields

A. a local minimum  
B. a local maximum  
C. a saddle point  
D. \( \nabla f(-1, -1) \neq 0 \)  
E. The Second Derivative Test gives no information at \((-1, -1)\)

11. Use the method of reversing the order of integration to compute
\[
\int_0^1 \int_0^2 e^{y^2} dydx.
\]

A. \( \frac{1}{4}(e^4 - 1) \)  
B. \( \frac{1}{2}(e^2 - 1) \)  
C. \( \frac{1}{6}(e^3 - 1) \)  
D. \( \frac{1}{2}(e^2 - e) \)  
E. \( \frac{1}{4}(e^2 - e) \)

12. A flat plate of constant density occupies the region in the \( xy \)-plane bounded by the curves \( x = 0 \) and \( x = \sqrt{1 - y^2} \). If \((\bar{x}, \bar{y})\) is the center of mass, then \( \bar{x} \) equals

A. \( \frac{2}{3\pi} \)  
B. \( \frac{1}{2} \)  
C. \( \frac{2}{\pi} \)  
D. \( \frac{3}{2\pi} \)  
E. \( \frac{4}{3\pi} \)
13. Find the volume of the solid whose base is the region in the $xy$-plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top of the solid is bounded by the plane $z = x + 4$.

A. $\frac{625}{12}$
B. $\frac{625}{11}$
C. $\frac{542}{13}$
D. $\sqrt{15} \pi$
E. $\sqrt{8} \pi$ 

14. Which of the following integrals equals the volume of the solid bounded by $x = 0$, $y = 0$, $z = 0$ and $2x + y + z = 4$.

A. $\int_0^4 \int_0^4 \int_0^2 1dxdydz$
B. $\int_0^2 \int_0^{1-2x} \int_0^{1-y} 1dzdydx$
C. $\int_0^4 \int_0^{2x} \int_0^{1-y} 1dzdydx$
D. $\int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} 1dzdydx$
E. $\int_0^2 \int_0^{1} \int_0^{1} 1dzdx dy$

15. Evaluate $\iint (x + 2y)dA$ where $R$ is the region of the plane bounded by $x + y = 2$, $x = y$, $y = 0$.

A. $\frac{1}{3}$
B. $\frac{5}{3}$
C. $\frac{7}{3}$
D. $\frac{11}{3}$
E. $\frac{14}{3}$
16. Let

\[ S: x = u - v, \ y = uv, \ z = u + v^2 \]

If \((0, b, 5)\) is a point on the tangent plane to \(S\) at \((0,1,2)\) on \(S\), then \(b = \)

A. \[3\]
B. 1
C. -2
D. 0
E. 2

17. Find \(\left( \frac{\partial w}{\partial y} \right)_x\) at \((w, x, y, z) = (4, 2, 1, -1)\) if

\[ w = x^2y^2 + yz - z^3, \quad x^2 + y^2 + z^2 = 6 \]

A. -1
B. 1
C. 3
D. \[5\]
E. 7

18. Consider the function \(f(x, y) = 2x^2 - 3xy + y^2\). Find two unit vectors such that the directional derivative of \(f\) at the point \((1,1)\) in these two directions is 1.

Answer: \((1,0)\) and \((0,-1)\)
19. Find cubic approximation of \( f(x, y) = \frac{1}{1 - x - y + xy} \) near the origin.

**Answer:** \( 1 + x + y + x^2 + xy + y^2 + x^3 + x^2y + xy^2 + y^3 \)

20. Find a equation for the tangent plane of

\[
\cos(\pi x) - x^2y + e^{xz} + yz = 4 \quad \text{at} \quad (0, 1, 2)
\]

**Answer:** \( 2x + 2y + z - 4 = 0 \)

21. Find absolute maximum and minimum values of

\[ f(x, y) = x^2 + 2y^2 - x \]

on the disc \( x^2 + y^2 \leq 1. \)

**Answer:** \( \text{max} = \frac{9}{4}, \text{min} = -\frac{1}{4} \)