Quiz 1

Problem 1
Solve the given initial-value problem

\[ \frac{dy}{dx} = 1 - \frac{\sin(x+y)}{\sin(y) \cos(x)}; \quad y(\pi/4) = \pi/4 \]

SOLUTION

\[ \frac{dy}{dx} = 1 - \frac{\sin(x+y)}{\sin(y) \cos(x)} \]

\[ = 1 - \frac{\sin(x) \cos(y) + \sin(y) \cos(x)}{\sin(y) \cos(x)} \]

\[ = -\frac{\sin(x) \cos(y)}{\sin(y) \cos(x)} \] (Common Mistake: missing the negative sign)

It is a separable differential equation.

\[ \frac{\sin(y)}{\cos(y)} \, dy = -\frac{\sin(x)}{\cos(x)} \, dx \]

\[ \int \frac{\sin(y)}{\cos(y)} \, dy = \int -\frac{\sin(x)}{\cos(x)} \, dx \]

\[ -\ln |\cos(y)| = \ln |\cos(x)| + C \]

\[ \frac{1}{\cos(y)} = C \cos(x) \]

Apply the initial condition:

\[ \frac{1}{\cos(\pi/4)} = C \cos(\pi/4) \]

\[ C = 2 \]

So, the solution is:

\[ 2 \cos(x) \cos(y) = 1 \]
**Problem 2**

Solve the given differential equation

\[ y' - y \tan(x) = 8 \sin^3(x) \]

**SOLUTION**

\[ I(x) = e^{\int -\tan(x) \, dx} = \cos(x) \]

Multiplying both sides of the differential equation by the integral factor:

\[
\frac{d}{dx}(\cos(x)y) = 8 \sin^3(x) \cos(x)
\]

\[
\cos(x)y = \int 8 \sin^3(x) \cos(x) \, dx
\]

\[
= \int 8 \sin^3(x) \, d\sin(x)
\]

\[
= 2 \sin^4(x) + C
\]

Therefore,

\[
y = \frac{2 \sin^4(x) + C}{\cos(x)} \neq \frac{2 \sin^4(x)}{\cos(x)} + C
\]