Quiz 1 Solutions

Show all of your work to receive full credit.

1. (a) Find an equation of the sphere with center \((-1, -2, 3)\) and radius 9.
   \(\text{Solution:}\) The sphere is the set of points at a distance of 9 units away from the point \((-1, -2, 3)\) and so we find that points on the sphere satisfy the distance equation \(\sqrt{(x - (-1))^2 + (y - (-2))^2 + (z - 3)^2} = 9\); the standard equation follows as \((x+1)^2 + (y+2)^2 + (z-3)^2 = 81\).

   (b) Use an equation to describe the intersection of this sphere with the \(yz\)–plane.
   \(\text{Solution:}\) Points on the \(yz\)–plane are characterized by having \(x\)–coordinate 0. Taking the standard equation from (a) and substituting in \(x = 0\) we find \((0+1)^2 + (y+2)^2 + (z-3)^2 = 81\). Simplifying, we get \((y+2)^2 + (z-3)^2 = 80\).

2. Determine whether the following planes are parallel, perpendicular or neither:
   \[ x + 4y - 3z = 1 \]
and
\[-3x + 6y + 7z = 0\]

*Solution:* Let \( \mathbf{n}_1 \) and \( \mathbf{n}_2 \) denote normal vectors to each of these planes respectively. We can see, from the coefficients of each plane equation, that one possible choice of these vectors is \( \mathbf{n}_1 = \langle 1, 4, -3 \rangle \) and \( \mathbf{n}_2 = \langle -3, 6, 7 \rangle \). The simplest case is to check for perpendicularity: if the dot product of the normal vectors is zero, it must follow that the normal vectors are at a right angle to each other, and so the planes themselves would necessarily be perpendicular. Hence we calculate: \( \mathbf{n}_1 \cdot \mathbf{n}_2 = (1)(-3) + (4)(6) + (-3)(7) = -3 + 24 - 21 = 24 - 24 = 0 \). Thus it turns out the planes are perpendicular and we can stop here without further calculation.