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Danielli, Donatella (1-PURD); Garofalo, Nicola (I-PADV-MM); Nhieu, Duy-Minh (RC-AST)
Isoperimetric and trace inequalities with respect to Carnot-Carathéodory metrics.
This note focuses on the problem of existence of traces for Sobolev spaces associated to a family of locally Lipschitz real vector fields \( \{X_1, \cdots, X_m\} \) in \( \mathbb{R}^n \). Denote by \( d \) the Carnot-Carathéodory metric associated to the system \( X_1, \cdots, X_m \). The vector fields must satisfy the following three conditions: (H1) The identity map from \( \mathbb{R}^n \) equipped with the Euclidean metric into \( \mathbb{R}^n \) equipped with the Carnot-Carathéodory metric is continuous. (H2) The balls of the Carnot-Carathéodory metric satisfy a doubling inequality with respect to the Lebesgue metric. (H3) There exists a weak-type Poincaré inequality with respect to the gradient associated to the \( X_1, \cdots, X_m \).
Examples are given by vector fields satisfying the H"ormander finite rank condition, by the so-called Grushin-Baouendi vector fields and by the (Lipschitz) vector fields associated to the subelliptic operators studied by Fefferman and Phong.

The main theorem states: Let \( f \) be a function whose weak gradient in the \( X_i \) directions has bounded \( L^p \) norm in an open set \( \Omega \). Then \( f \) has an \( L^p \) restriction to the boundary of \( \Omega \) when the surface balls have the correct rate of growth. In the particular case of the Heisenberg group this happens for every \( C^2 \) set.
Several extensions and applications are mentioned in the paper, together with an outline of the proofs.

{For the entire collection see 99b:00019} Luca Capogna (1-AR)