1. Let $A$ be a $3 \times 3$ matrix such that 

$$AA^T = \begin{pmatrix} 5 & 2 & 5 \\ 2 & 2 & 1 \\ 6 & 1 & 10 \end{pmatrix}.$$ 

Find $\det(A)$.
2. A metal object is heated to $200^\circ \text{C}$ and then placed in a room to cool. The temperature of the room is held constant at $20^\circ \text{C}$. After 10 minutes, the object temperature is $100^\circ \text{C}$. How long will it take the object to cool to $25^\circ \text{C}$?
3. Find the solution of the initial value problem

\[ y' = -y + e^x y^2, \quad y(-1) = 1. \]
4. Consider the equation

\[ x^m y^2 y' + \alpha x^3 y^n = 0. \]

(a) Find the values of the constants \( \alpha, m, \) and \( n \) for which the equation is exact.

(b) Solve the equation using the values of \( \alpha, m, \) and \( n \) found in part (a).
5. Determine all values of $k$ for which the system

\begin{align*}
    x_1 - kx_2 + k^2x_3 &= 0 \\
    x_1 + kx_3 &= 0 \\
    x_2 - x_3 &= 0
\end{align*}

has (a) infinite solutions; (b) no solutions; (c) exactly one solution.
6. A tank initially contains 120 liters of pure water. A mixture containing a concentration of 2 g/l of salt enters the tank at a rate of 2 l/min. The well-stirred mixture leaves the tank at the same rate. What is the amount of salt in the tank after one hour?
7. (a) Use reduced row-echelon reduction to find $A^{-1}$

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -2 & -1 & 1 \\ 3 & 0 & 3 \end{pmatrix}.$$ 

(b) Use part (a) to solve the system $Ax = b$, where $b = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$. 