Math 460: Homework # 4. Due Thurs. Feb 13

1. Use Geometer’s Sketchpad to construct a triangle, along with the following

   (a) its circumcenter (labeled $O$)
   (b) its incenter (labeled $I$)
   (c) its orthocenter (labeled $H$)
   (d) its centroid (labeled $G$)
   (e) the line through $O$ and $H$.

   Hide all the lines used in constructions (a)-(d). Print out a copy, then change the shape of the triangle and print another copy. The line through $O$ and $H$ has a special property that should be obvious from your pictures—what is it? (You do not need to prove anything for this problem.)

2. (see Figure 1). Give the proof of Theorem 24 for Case (ii). Given: $M, N,$ and $P$ are the midpoints of $AB, AC,$ and $BC$ respectively, $MX \perp AB,$ and $NX \perp AC$. To prove: $X$ is on the perpendicular bisector of $BC$. (Hint: Use the same strategy that was used in the course notes for Case (i)).

   ![Figure 1](image)

3. (See Figure 2.) Give the proof of Theorem 24 for Case (iii). Given: $M$ and $N$ are the midpoints of $AB$ and $AC$, $MX \perp AB$, $NX \perp AC$, and $X$ is on $BC$. To prove: $X$ is on the perpendicular bisector of $BC$. 

   ![Figure 2](image)
4. Let $ABC$ be an equilateral triangle and let $P$ be a point which is outside the triangle but in the interior of $\triangle ABC$. Let $a$, $b$, and $c$ be the distances from $P$ to $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$ respectively. Let $h$ be the height of triangle $ABC$. To prove: $a - b + c = h$.

5. (See Figure 3.) Given: $AD = BC$, $AC = BD$, $AK = BN$. To prove: $KG = NH$.

6. (See Figure 4.) Given: The lines that look straight are straight, $\angle D = \angle 1$, $KM = TM = CM$. To prove: $AD = BC$. 

![Figure 2](image1.png)

![Figure 3](image2.png)

![Figure 4](image3.png)
Figure 4
7. (See Figure 5.) Given: \(ABCD\) is a parallelogram, \(\angle 1 = \angle 2, \angle 3 = \angle 4\), \(EF\) is parallel to \(AD\). To prove: \(AF = FB\).

8. (See Figure 6.) Given that \(AD\) is parallel to \(BE\) and \(BE\) is parallel to \(CF\), prove that \(\frac{AB}{AC} = \frac{DE}{DF}\).