1. a) is false (take 2 times 2 Jordan cell, for example).
   b) False (take two unit matrices).
   c) True, because \((A + B)^H = A^H + B^H\).
   d) True, (think of the geometric meaning of orthogonality).
   e) False.

2. a) FALSE! (Take the unit matrix. Then ALL vectors are eigenvectors, so EVERY basis consists of eigenvectors! I warned you about this in class.)
   b) True (all eigenvalues are real and have absolute value 1. So they are all \(\pm 1\). A diagonal matrix with such eigenvalues certainly satisfies \(D^2 = I\), so \(A\) satisfies this as well because \(A\) is diagonalizable).
   c) False (take a triangular matrix with all eigenvalues different, for example).
   d) False (take a Jordan cell with zero eigenvalue).
   e) True (it is diagonalizable and the diagonal matrix is \(I\), so \(A = QIQ^{-1} = I\)).

3. The eigenvalues are \(1 \pm i\). Raising them to 100-s power:
   \[(1 \pm i)^{100} = \sqrt{2}^{100} \exp(\pm 100\pi i/4) = -2^{50}.
   
   This is the regular way to do this. But for this special number there is another simple way:
   \[(1 \pm i)^{100} = (1 \pm i)^4^{25} = -4^{25} = -2^{50}.
   
   So \(A\) is diagonalizable (because it has two different eigenvalues) and its diagonal form is \(-2^{50}I\). This commutes with everything, so \(A^{100} = C(-2^{50}I)C^{-1} = -2^{50}I\).

4. First solution. The only eigenvalue is 0, and there is only one-dimensional space of eigenvectors. Thus \(B\) is similar to the Jordan cell
   \[J = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \text{and} \quad e^{Jt} = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}.
   
   Take \((1, 1)^T\) as an eigenvector of \(B\). Then a generalized eigenvector will be \((1, 0)\). So
   \[e^{Bt} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 1 + t & -t \\ t & 1 - t \end{pmatrix}.
   
   1
Second, more elegant solution. Notice that $B^2 = 0$. Thus by definition of the exponential:

$$e^{Bt} = I + Bt = \begin{pmatrix} 1 + t & -t \\ t & 1 - t \end{pmatrix}.$$ 

5. a) $A^T = \lambda A$ implies that $A = (A^T)^T = \lambda^2 A$ which is possible with $A \neq 0$ only when $\lambda = \pm 1$. Eigenvectors for $\lambda = 1$ are symmetric matrices, and eigenvectors for $\lambda = -1$ are skew-symmetric ones.

   b) The dimensions of these spaces are $n(n+1)/2$ and $n(n-1)/2$, respectively; if $n = 2$ then they are 3 and 1.

   c) $A$ is diagonalizable because it has $n(n+1)/2 + n(n-1)/2 = n^2$ linearly independent eigenvectors. (When $n = 2$ we have $3 + 1 = 4$, which is the dimension of the whole space of $2 \times 2$ matrices.)

   d) In my favorite basis it is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$ 

6. $C$ is symmetric so it cannot be similar to $J$. (This observation alone was worth 3 points). To investigate the other matrices, look at the eigenspace corresponding to $\lambda = 1$. For $J$ it is of dimension 1. For $A$ it is of dimension 2, so $A$ is not similar to $J$. Thus the answer is $A$ and $C$.

Remark. This was NOT a multiple choice question! partial credit was given. However, as it is always the case with partial credit questions, I was looking at your arguments. So sometimes no credit was given if the arguments were completely wrong, no matter what the answer was. For example, computing determinants, traces etc. was completely irrelevant, because I told you in advance what the eigenvalues are. So all 4 matrices have determinant 2 and trace 4.

7. $(-1)^{n+1}$. 