1. 
\[ f_{|X|}(t) = \frac{d}{dt} \int_{-t}^{t} f_X(x)dx = f_X(t) + f_X(-t), \quad t \geq 0, \]
and \( f_{|X|}(t) = 0 \) if \( t < 0 \).

(Using the ordinary change of the variable formula here is wrong: first \(|x|\) is not differentiable, second, it is not one-to-one.)

2. \( \phi(t) = \sqrt{t} \) (Change of the Variable formula).

3. Multiplication by \(-2\) results in multiplication of the mean by \(-2\) and variance by \((-2)^2 = 4\), and the result has normal distribution. Sum of two independent normal variables is normal, so the answer is \( n(\mu_1 - 2\mu_1, \sigma_2^2 + 4\sigma_1^2) \).

4. Joint density of \( X \) and \( Y \) is \( \lambda^2 \exp(-\lambda(x+y)) \), for \( x > 0 \) and \( y > 0 \), and zero otherwise. By the Change of the Variable formula, the joint density of \( X \) and \( Z \) is
\[ f_{X,Z}(x,z) = \lambda^2 \exp(-\lambda x + (z-x)) = \lambda^2 \exp(-\lambda z) \]
for \( x > 0 \) and \( 0 < x < z \), and zero otherwise. (Jacobian determinant is 1). This answers a).

The conditional density of \( X \) for given \( Z = z \) is
\[ \frac{f_{X,Z}(x,z)}{\int_0^z f_{X,Z}(x,z)dx} = \frac{1}{z}, \quad 0 < x < z. \]

so, for fixed \( Z = z \), \( X \) is uniformly distributed on \((0, z)\).

5. 
\[ F_R(t) = c \int \int_{x^2+y^2 \leq t} e^{-(x^2+y^2)/2\sigma^2} dxdy, \]
where \( c \) is some constant. Passing to polar coordinates we obtain
\[ F_R(t) = 2\pi c \int_0^t e^{-r^2/2\sigma^2} rdr = c_1(1 - e^{-t^2/2\sigma^2}). \]

Now it is clear that \( c_1 = 1 \), otherwise the RHS is not a distribution function. (Or you could use the correct value of \( c \) from the very beginning). Differentiating we obtain
\[ f_R(t) = \frac{t}{\sigma^2} e^{-t^2/2\sigma^2}, \]
which is a Gamma density, by the way.

An alternative method is to refer to the theorems we had in class: \(X^2\) and \(Y^2\) have some Gamma densities and then you can use the theorem on the sum of independent variables with gamma densities. But to get credit you had to use these theorems CORRECTLY. So if you are not 100% sure what a theorem exactly says, better do computation. For a computation with a mistake you may get partial credit, but for the incorrect use of a theorem this is less likely.

6. \(EZ = E(X - \rho Y) = 0\), this answers a). Now, using this, we obtain

\[
\text{Var}Z = EZ^2 = EX^2 - 2\rho E(XY) + \rho EY^2.
\]

But \(E(XY) = \rho\) and \(EX^2 = EY^2 = 1\) by the conditions of this problem, so \(\text{Var}Z = 1 - \rho^2\). This answers b). Now, using that expectations of \(X\) and \(Z\) are zero,

\[
\text{Cov}(Y, Z) = E(YZ) = E(Y(X - \rho Y)) = EXY - \rho EY^2 = 0.
\]

7. One of the variables is discrete while another has density, so (unless you have experience with Stieltjes integrals, which I do not assume), it is better to compute using only definitions. Using independence we obtain

\[
P(X + Y \leq t) = P(X \leq t, Y = 0) + P(X + 1 \leq t, Y = 1)
\]

\[
= (1/2) (P(X \leq t) + P(X \leq t - 1)) = F_X(t) + F_X(t - 1),
\]

so

\[
f_{X+Y}(t) = \frac{1}{2} (f_X(t) + f_X(t - 1)) = \begin{cases} 1/2, & 0 \leq t \leq 2, \\ 0, & \text{otherwise} \end{cases}
\]

(The value of a density at one point \(t = 1\) is not irrelevant.)

8. If \(Z\) is the length of the left piece, then \(Z\) is uniformly distributed on \((0, 1)\), and \(X = \min\{Z, 1 - Z\}\), and \(Y = \max\{Z, 1 - Z\}\). So

\[
EX = \int_0^1 \min\{z, 1 - z\}dz = \int_0^{1/2} zdz + \int_{1/2}^1 (1 - z)dz = \frac{1}{4}.
\]
This answers the first question. For the second question we have

\[ E(Y/X) = \int_0^1 \frac{\max\{z, 1 - z\}}{\min\{z, 1 - z\}} \, dz = \int_0^{1/2} \frac{1 - z}{z} \, dz + \ldots, \]

the integral is divergent, so the expectation does not exist.