Wandering domains of entire functions

A. Eremenko

April 4, 2015

Let $f$ be an entire function, which is not of the form $az+b$. Denote by $f^n$ the $n$-th iterate of $f$, that is $f^n = f \circ \ldots \circ f$ $n$ times. Let $F$ be the maximal open set in the complex plane, where the iterates $\{f^n : n = 0, 1, 2, \ldots\}$ of $f$ form a normal family.

Suppose that $D$ is a component of $F$. Consider the set $L$ of all limit functions of the family $\{f^n\}$, and suppose that all these limit functions are constant. There are examples where $L$ is infinite.

**Question.** Can $L$ be infinite and bounded?

Here is a restatement of the question in modern terminology. A component $D$ of the set $F$ is called wandering if $f^n(D) \cap f^m(D) = \emptyset$, for all integers $n > m \geq 0$. It is known that all limit functions of the family $\{f^n\}$ in a wandering component $D$ are constant. Furthermore, if the set $L$ of limit functions in a component $D$ of $F$ consists only of constants, and $L$ is infinite, then $D$ is a wandering domain. The question is whether a subdomain $D_1 \subset D$ of a wandering domain can wander on a bounded subset of the plane.