We introduce the following collection of sets
\[ \mathcal{E} = \{ E \subset \mathbb{H}^n \mid E \text{ satisfies (i) - (ii)} \}, \]
where
1. \(|E \cap \mathbb{H}^n_+| = |E \cap \mathbb{H}^n_-|\);
2. There exist \(R > 0\) and \(C^1\) functions \(u, v : [0, R] \to \mathbb{R}\) satisfying \(u(R) = v(R)\), such that \(\partial E \cap \mathbb{H}^n_+ = \{(z, t) \mid t = u(|z|^2/4)\}\) and \(\partial E \cap \mathbb{H}^n_- = \{(z, t) \mid t = v(|z|^2/4)\}\).

**Theorem 0.1.** Given \(V > 0\), the variational problem
\[ \min\{ P_X(E; \mathbb{H}^n) \mid E \in \mathcal{E}, \ |E| = V \} \]
has a unique solution \(E_o \in \mathcal{E}\). Furthermore, \(\partial E_o\) is given explicitly as the graph of the function
\[ t = \pm \left\{ \frac{1}{4} |z| \sqrt{R^2 - |z|^2} - \frac{R^2}{4} \tan^{-1} \left( \frac{|z|}{\sqrt{R^2 - |z|^2}} \right) + \frac{\pi R^2}{8} \right\}, \quad |z| \leq R. \]
The sign \(\pm\) depends on whether we are considering \(\partial E_o \cap \mathbb{H}^n_+\), or \(\partial E_o \cap \mathbb{H}^n_-\). Finally, the set \(E_o\) is of class \(C^2\) near its two characteristic points \(\left(0, \pm \frac{\pi R^2}{8}\right)\), and \(S = \partial E_o\) has positive constant \(X\)-mean curvature given by
\[ H_X = \frac{Q-2}{Q-1} \frac{1}{R}. \]