Theorem: $\iota_*([F]) = f^*([B,\partial B]) \cap [E,\partial E]$.

Here $f: E \to B$ is a smooth map between two compact manifolds with boundary so that $f(\partial E) \subset \partial B$. Let F be the fibre of some regular point $b \in B$. Then $[F]$ is the unique fundamental class of $[F]$ given by the fundamental classes $[B,\partial B]$ and $[E,\partial E]$ as follows.

Let D be a small disk about b, so that $f^{-1}(D) = D \times F$. This D can be found since b is a regular value. Now the orientation on B induces an orientation on D and the orientation on E induces the orientation on $f^{-1}(D)$. Then there is a unique class $[F] \in H_r(F;\mathbb{Z})$ which induces an orientation on $D \times F$ consistent with the orientations of D and F. (Think of the case when F is disconnected to appreciate the full impact of this statement.)

Now by Proposition 2 of [Partial - Transfer]

$f^*([\overline{\partial B}]) \cap [\partial E] = i_*([F])$ where F is a fibre of a regular point of $f: \partial E \to \partial B$, and the class $[F]$ is chosen as above on ∂E and ∂B.

Let $j: \partial E \to E$ denote the inclusion. Then

$$j_* i_*([F]) = j_*\left(f^*[\overline{\partial B}] \cap [\partial E]\right)$$

$$= (-1)^F \left(\delta^* f^* [\overline{\partial B}] \cap [E, \partial E]\right)$$

(Dold 9.1)

$$= (-1)^F \left(f^* \delta^* [\overline{\partial B}]\right) \cap [E, \partial E]$$

$$= (-1)^F \left(f^* [\overline{B, \partial B}]\right) \cap [E, \partial E]$$

Now $j_* i_*([F]) = i'_*([F])$ where i' is the inclusion of F into E. But the class $[F]$ is defined by $[\partial E]$ and $\overline{[\partial B]}$. The unique orientation class defined by $[E, \partial E]$ and $[B, \partial B]$ is $(-1)^F$ times $[F]$. Using these remarks and changing the notation, we see that

$$i_*[F] = f^*(\overline{[B, \partial B]}) \cap [E, \partial E].$$