NOTE ON GAUSS-BONNET

by

Daniel Henry Gottlieb

Several geometers have pointed out that I have not given due credit to Allendoefer and Weil in my recent article on the Gauss-Bonnet Theorem, All the Way with Gauss-Bonnet and the Sociology of Mathematics, Monthly 103,(1996), 457-469.

As described on page 464 of my article, Hopf asked for an intrinsic proof and generalization of the even dimensional case of his Satz VI, the topological Gauss-Bonnet theorem. In 1940, Allendoerfer and Fenchel independently found the correct formula, which today is called the Gauss-Bonnet-Chern theorem, but only for Riemannian submanifolds in Euclidean space. However in 1943, Allendoerfer and Andre Weil proved that in fact it held for all Riemannian manifolds. This is clearly stated in Theorem I on page 101 of C.B. Allendoerfer and Andre Weil, Amer. Math. Soc. Trans. 53 (1943), 101-129. This is the complete Gauss-Bonnet-Chern theorem. But their proof involved local embeddings, and hence was not intrinsic.

Later Weil told Chern about the lack of an intrinsic proof for the Gauss-Bonnet-Chern Theorem. Chern found his famous proof in short order, and published it in the Annals in 1944 with a title that accurately reflected his contribution: "A simple intrinsic proof of the Generalized Gauss-Bonnet theorem". The Nash embedding theorem which states that every Riemannian manifold can be found as a submanifold of Euclidean space was proved in the 1950's. When combined with the Allendoerfer-Fenchel result of 1940, Nash's theorem renders Allendoerfer-Weil's step superfluous. Chern's proof and Nash's theorem probably helped create the widespread misconception obtaining today about the credit due to Allendoerfer and

Weil.

Daniel Henry Gottlieb
Department of Mathematics
Purdue University
West Lafayette, Indiana, 47907
Purdue FAX: 317-494-0549
Purdue phone: 317-49-41951
Home Phone: 317-463-2816
WWW Home Page URL: http://www.math.purdue.edu:80/~gottlieb/
E-mail: gottlieb@math.purdue.edu
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