Becker-Gottlieb Transfer

The Becker-Gottlieb transfer is an S-map from the base to the total space of a fibration. It thus induces homomorphisms on homology and cohomology which are also called Becker-Gottlieb transfers.

Suppose that $E \xrightarrow{p} B$ is a Hurewicz fibration whose fibre F is homotopy equivalent to a compact CW complex F. The Becker-Gottlieb transfer is an S-map $\tau : B \to E$, which means that there is a map $\tau : \Sigma^N B \to \Sigma^N E$ defined between the N-th suspensions of B and E for some N. Thus τ induces a homomorphism τ_* on any homology theory and τ^* on any cohmology theory.

We obtain striking relations on homology and cohomology respectively:

$$p_* \circ \tau_* = \chi(F), \quad \tau^* \circ p^* = \chi(F) \tag{1}$$

where $\chi(F)$ denotes multiplication by the Euler-Poincare number of F.

The Becker-Gottlieb transfer was discovered in the mid seventies. It generalizes the transfer for finite covering spaces (F is a finite set of points), which had been well-known since the forties and which was a generalization of a group theory transfer from the twenties.

Since the discovery of the Becker-Gottlieb transfer, other transfers have been discovered which satisfy equations (1) with the Euler-Poincare number replaced by another elementary topological invariant. Thus

$$p_* \circ \tau_* = k, \quad \tau^* \circ p^* = k \tag{2}$$

where k denotes multiplication by an integer k, which can be a Lefschetz number, a coincidence number, a fixed point index, a vector field index, or an intersection number. For any map $p : X \to Y$, the greatest common divisor of all the k's associated to a transfer is the Brouwer degree of p when X and Y are manifolds of the same dimension. Some of these transfers are induced by S-maps, and others are not.

Several mathematicians contributed to the discovery of these other transfers, most of which either generalize some or all of the Becker-Gottlieb transfer. Albrecht Dold's fixed point transfer was discovered independently of the Becker-Gottlieb transfer, which it generalizes. It is sometimes called the Becker-Gottlieb-Dold transfer.

Reference:

James C. Becker and Daniel H. Gottlieb, Vector fields and Transfers, Manuscripta Mathematica, 72(1991), pp.111-130.