1. A participant in a pension plan is subject to three decrements: (1) is retirement, (2) is death, (3) is other termination. All terminations have a constant force of decrement as follows:

i. \( \mu_x^{(1)} = 0.05 \)
ii. \( \mu_x^{(2)} = 0.01 \)
iii. \( \mu_x^{(3)} = 0.10 \)

Calculate the following items:

a. \( \mu_x^{(r)} \)
b. \( tP_x^{(r)} \)
c. \( f_{r,j}(t) \) for \( j = 1, 2, \text{and } 3 \)
d. \( tq_x^{(j)} \) for \( j = 1, 2, \text{and } 3 \)
e. \( tq_x^{(r)} \)
f. \( \infty q_x^{(j)} \) for \( j = 1, 2, \text{and } 3 \)
g. \( q_x^{(3)} \)
h. \( q_x^{(3)}_{x+1} \)
i. \( 2q_x^{(3)} \)
j. \( 2|q_x^{(3)} \)
k. \( tP_x^{(j)} \) for \( j = 1, 2, \text{and } 3 \)
l. \( tq_x^{(j)} \) for \( j = 1, 2, \text{and } 3 \)
2. Your professor is 54 years old. My teaching career is subject to two decrements as follows:

   i. Decrement 1 is mortality. The associated single decrement table for future mortality follows De Moivre’s law with $\omega = 50$.
   ii. Decrement 2 is retirement. The associated single decrement table for future retirement follows De Moivre’s law with $\omega = 20$.

Calculate the following items:

   a. $\mu_x^{(j)}(t)$ for $j = 1$ and 2
   b. $\mu_x^{(r)}(t)$
   c. $tP_x^{(j)}$ for $j = 1$ and 2
   d. $tP_x^{(r)}$
   e. $f_T(t, j)$ for $j = 1$ and 2
   f. $tq_x^{(j)}$ for $j = 1$ and 2
   g. $tq_x^{(r)}$
   h. $tq_x^{(j)}$ for $j = 1$ and 2
   i. $q_x^{(1)}$
   j. $q_x^{(1)}$
   k. $q_{x+1}^{(1)}$
   l. $q_{x+1}^{(1)}$
   m. $2q_x^{(2)}$
   n. $2q_x^{(2)}$
   o. $2q_x^{(2)}$
3. (Spreadsheet) One thousand whole life policies are issued to males (20). The policies are subject to two decrements as follows:

   i. Mortality (1) where \( q_x^{(2)} \) is the mortality rate in the Excel File on Class Website
   ii. Lapse (2) where \( q_x^{(2)} \) is the lapse rate in the following table

<table>
<thead>
<tr>
<th>( x )</th>
<th>( q_x^{(2)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.20</td>
</tr>
<tr>
<td>21</td>
<td>0.10</td>
</tr>
<tr>
<td>22</td>
<td>0.08</td>
</tr>
<tr>
<td>23</td>
<td>0.07</td>
</tr>
<tr>
<td>24</td>
<td>0.06</td>
</tr>
<tr>
<td>25+</td>
<td>0.05</td>
</tr>
</tbody>
</table>

In a spreadsheet complete the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( q_x^{(1)} )</th>
<th>( q_x^{(2)} )</th>
<th>( l_x^{(r)} )</th>
<th>( d_x^{(1)} )</th>
<th>( d_x^{(2)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Using the table created in Problem 3, calculate the following:

   i. \( 5\overline{p}_25^{(r)} \)
   ii. The probability of termination by lapse between the end of the 5\(^{th}\) duration and the end of the 10\(^{th}\) duration.
   iii. The number of policies still in force at attained age 65.
   iv. \( 10\overline{q}_{35}^{(1)} \)
   v. \( 10d_{45}^{(r)} \)
5. You are given:
   i. \( q_{x}'(1) = 0.200 \)
   ii. \( q_{x}'(2) = 0.080 \)
   iii. \( q_{x}'(3) = 0.125 \)

   Assuming that each decrement is uniformly distributed over each year of age in the associated single decrement table, calculate \( q_{x}'(1) \).

6. You are given:
   i. \( q_{x}'(1) = 0.200 \)
   ii. \( q_{x}'(2) = 0.080 \)
   iii. \( q_{x}'(3) = 0.125 \)

   Assuming that each decrement in the multiple decrement table is uniformly distributed over each year of age, calculate \( q_{x}'(1) \).

7. You are given the following for a double decrement table:
   i. \( q_{x}'(1) = 0.200 \)
   ii. \( q_{x}'(2) = 0.080 \)

   Assuming that each decrement in the multiple decrement table is uniformly distributed over each year of age, calculate \( 0.4q_{x}'(1) + 0.4 \).

8. You are given:
   i. \( q_{x}'(1) = 0.200 \)
   ii. \( q_{x}'(2) = 0.080 \)

   Decrement (1) is uniformly distributed over the year. Decrement (2) occurs at time 0.6.

   Calculate \( q_{x}'(2) \).

9. For a double decrement table with \( l_{40}' = 2000 \):

<table>
<thead>
<tr>
<th></th>
<th>( q_{x}'(1) )</th>
<th>( q_{x}'(2) )</th>
<th>( q_{x}'(1) )</th>
<th>( q_{x}'(2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.24</td>
<td>0.10</td>
<td>0.25</td>
<td>y</td>
</tr>
<tr>
<td>41</td>
<td>--</td>
<td>--</td>
<td>0.20</td>
<td>2y</td>
</tr>
</tbody>
</table>

   Calculate \( l_{42}' \)
10. For iPhones, the phone may cease service for mechanical failure or for other reasons (lost, stolen, dropped in a pitcher of beer, etc). You are given the following double decrement table:

<table>
<thead>
<tr>
<th>Year of Service</th>
<th>For an iPhone at the beginning of the year of service, probability of</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mechanical Failure</td>
<td>Failure for Other Reasons</td>
<td>Survival through the year of service</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.30</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>--</td>
<td>0.40</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>--</td>
<td>--</td>
<td>0.20</td>
</tr>
</tbody>
</table>

You are also given:

i. The number of iPhones that terminate for other reasons in year 3 is 40% of the number of iPhones that survive to the end of year 2.

ii. The number of iPhones that terminate for other reasons in year 2 is 80% of the number of iPhones that survive to the end of year 2.

Calculate the probability that an iPhone will cease to function due to mechanical failure during the three year period following its entry into service.

11. A whole life policy pays a death benefit of 100,000 upon death from an automobile accident. The policy also pays 50,000 upon death from any other accident or 25,000 upon death from natural (non-accidental) causes.

You are given:

i. $\delta = .10$

ii. $\mu_x^{(1)} = 0.02$ where (1) is death in an automobile accident.

iii. $\mu_x^{(2)} = 0.04$ where (2) is death in any other accident.

iv. $\mu_x^{(3)} = 0.01$ where (3) is death in natural causes.

v. Death Benefits are payable at time of death.

Calculate the single benefit premium.
12. A fully discrete 3 year term pays a benefit of 1000 upon any death. It pays an additional 1000 (for a total of 2000) upon death from accident. You are given:

<table>
<thead>
<tr>
<th>x</th>
<th>$q_x^{(1)}$</th>
<th>$q_x^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.030</td>
<td>0.010</td>
</tr>
<tr>
<td>21</td>
<td>0.025</td>
<td>0.020</td>
</tr>
<tr>
<td>22</td>
<td>0.020</td>
<td>0.030</td>
</tr>
</tbody>
</table>

Decrement (1) is death from accidental causes while decrement (2) is death from non-accidental causes.

The annual effective interest rate is 10%.

Calculate the level annual benefit premium for this insurance.

13. Norris Life Insurance Company sells a fully discrete whole life policy with a benefit of 1 million to (65). The policy has level annual premiums. You are given the following assumptions:

a. Mortality follows the mortality in the Illustrative Life Table.
b. Interest is 6% per annum.
c. Expenses at the beginning of each year are as follows:
   1. Per policy expense is $100 in the first year and $40 per policy for each year thereafter;
   2. Percent of premium expenses are 50% in the first year and 3% thereafter;
   3. Per 1000 expense of $1.00 in the first year and $0.10 thereafter; and
   4. $200 per policy per claim.

i. Calculate the annual benefit premium.
ii. Calculate the annual expense loaded premium.
iii. Calculate the level annual Expense Premium.
iv. Calculate the benefit reserve at the end of the 10th year.
v. Calculate the expense reserve at the end of the 10th year.
vi. Calculate the total reserve at the end of the 10th year.
vii. Calculate the Asset Share at the end of the first year.
viii. Calculate the Asset Share at the end of the second year.
14. Norris Life also sells a three year term insurance policy with a benefit of 10,000 to (x). Annual premiums are payable for three years. Death benefits are assumed to be paid at the end of the year. You are given the following:

a. All expenses occur at the beginning of the year.
b. Interest is 8%.
c.

<table>
<thead>
<tr>
<th>Year</th>
<th>Mortality</th>
<th>Per Policy Expense</th>
<th>Percent of Premium Expense</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.010</td>
<td>130</td>
<td>20%</td>
</tr>
<tr>
<td>2</td>
<td>0.015</td>
<td>30</td>
<td>8%</td>
</tr>
<tr>
<td>3</td>
<td>0.020</td>
<td>30</td>
<td>8%</td>
</tr>
</tbody>
</table>

i. Calculate the expense loaded premium.

ii. Complete the following table:

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( iV^b )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( iV^e )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( i\text{AS} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

15. Norris Life also sells a three year term insurance policy with a benefit of 10,000 to (x). Annual premiums are payable for three years. Death benefits are assumed to be paid at the end of the year. Lapses are also assumed to occur at the end of the year. You are given the following:

i. All expenses occur at the beginning of the year.
ii. Interest is 8%.
iii.

<table>
<thead>
<tr>
<th>Year</th>
<th>Mortality</th>
<th>Lapses</th>
<th>Per Policy Expense</th>
<th>Percent of Premium Expense</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.010</td>
<td>0.20</td>
<td>130</td>
<td>20%</td>
</tr>
<tr>
<td>2</td>
<td>0.015</td>
<td>0.10</td>
<td>30</td>
<td>8%</td>
</tr>
<tr>
<td>3</td>
<td>0.020</td>
<td>0.05</td>
<td>30</td>
<td>8%</td>
</tr>
</tbody>
</table>

iv. The expense loaded premium is 231.01.
v. There is no withdrawal benefit.

Calculate the Asset Share at \( t = 0, 1, 2, \) and 3
16. You are given the following for a 10,000 fully discrete whole life policy with level annual premiums of 250:
   ii. \( 3AS = 500 \)
   iii. \( 4AS = 662.20 \)
   iv. Per policy expenses of 60 for the first year and 10 for all other years.
   v. Percent of Premium expenses of 40% in the first year and 10% in other years.
   vi. \( q_x^{(d)} = 0.01 \) where (d) indicates death
   vii. \( q_x^{(w)} = 0.05 \) where (w) indicates withdrawals which occur at the end of the year
   viii. \( 3CV = 100 \)
   ix. \( 4CV = 200 \)

Calculate the interest rate used.

17. (SOA November 1987) For a single premium continuous whole life issued to (x) with a face amount of \( F \), you are given:
   x. \( \bar{A}_x = 0.2 \)
   xi. Percent of premium expenses are 8% of the expense-loaded premium.
   xii. Per policy expenses are 75 at the beginning of the first year and 25 at the beginning of each subsequent year.
   xiii. Claim expenses are 15 at the moment of death.
   xiv. \( i = 0.05 \)
   xv. Deaths are uniformly distributed over each year of age.
   xvi. The expense loaded premium is expressed as \( gF + h \).

Calculate \( g \) and \( h \).
18. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:
   a. In Good Standing
   b. Out of Favor
   c. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

<table>
<thead>
<tr>
<th></th>
<th>In Good Standing</th>
<th>Out of Favor</th>
<th>Dead</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Good Standing</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Out of Favor</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>Dead</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Calculate the probability that a person In Good Standing now will be Out of Favor at the end of the fourth year.

19. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:
   a. In Good Standing
   b. Out of Favor
   c. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

<table>
<thead>
<tr>
<th></th>
<th>In Good Standing</th>
<th>Out of Favor</th>
<th>Dead</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Good Standing</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Out of Favor</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>Dead</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

At the beginning of the year, there are 1000 employees In Good Standing. All future states are assumed to be independent.

i. Calculate the expected number of deaths over the next four years.
ii. Calculate the variance of the number of the original 1000 employees who die within four years.
20. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:
   a. In Good Standing
   b. Out of Favor
   c. Dead

The following transitional probabilities indicate the probabilities of moving between the three status in a given year:

<table>
<thead>
<tr>
<th></th>
<th>In Good Standing</th>
<th>Out of Favor</th>
<th>Dead</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Good Standing</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Out of Favor</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>Dead</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The Italian Life Insurance Company issues a special 4 year term insurance policy covering employees of OCI. The policy pays a death benefit of 10,000 at the end of the year of death.

Assume that the interest rate is 25% (remember who we are dealing with).

i. Calculate the actuarial present value of the death benefit for an employee who is In Good Standing at the issue of the policy.

ii. Calculate the annual benefit premium (paid at the beginning of the year by those in Good Standing and those Out of Favor) for an employee who is In Good Standing at the issue of the policy.

iii. Calculate the reserve that Italian Life should hold at the end of the second year for a policy that was issued to an employee who was In Good Standing.

iv. Calculate the actuarial present value of the death benefit for an employee who is Out of Favor at the issue of the policy.

v. For an employee who is Out of Favor when the policy is issued, the annual contract premium payable at the beginning of each year that the employee is not Dead is 3500. Calculate the actuarial present value of the expected profit for Italian Life. The actuarial present value of the expected profit is the actuarial present value of the contract premiums less the actuarial present value of the death benefits.
21. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:

   a. In Good Standing
   b. Out of Favor
   c. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

<table>
<thead>
<tr>
<th></th>
<th>In Good Standing</th>
<th>Out of Favor</th>
<th>Dead</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Good Standing</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Out of Favor</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>Dead</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The Italian Life Insurance Company issues a special four year annuity covering employees of OCI. The annuity pays a benefit of 100,000 at the end of a year if the employee is In Good Standing at the end of the year. It pays a benefit of 50,000 if an employee is Out of Favor at the end of a year. No benefit is paid if the employee is Dead at the end of a year.

Assume that the interest rate is 25% (remember who we are dealing with).

i. Calculate the actuarial present value of the annuity benefit for an employee who is In Good Standing at the issue of the policy.

ii. Calculate the annual benefit premium (paid at the beginning of the year by those in Good Standing and those Out of Favor) for an employee who is In Good Standing at the issue of the policy.

iii. Calculate the reserve that Italian Life should hold at the end of the second year for a policy that was issued to an employee who was In Good Standing.
22. Animals species have three possible states: Healthy (row 1 and column 1 in the matrices), Endangered (row 2 and column 2 in the matrices), and Extinct (row 3 and column 3 in the matrices). Transitions between states are modeled as a non-homogeneous Markov chain with transition matrices $Q_i$ as follows:

$$Q_0 = \begin{pmatrix} 0.80 & 0.20 & 0 \\ 0 & 0.75 & 0.25 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_1 = \begin{pmatrix} 0.90 & 0.10 & 0 \\ 0.20 & 0.70 & 0.10 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.25 & 0.70 & 0.05 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_i = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.3 & 0.70 & 0 \end{pmatrix}$$

Calculate the probability that a species endangered at time 0 will become extinct.

23. The number of home runs hit in baseball game are distributed as a Poisson distribution with home runs hit at a rate of 3 per nine innings.
   i. Calculate the expected number of home runs in a nine inning game.
   ii. Calculate the variance of the number of home runs in a nine inning game.
   iii. Calculate the probability that zero home runs will be hit in a nine inning game.
   iv. Calculate the probability that five home runs will be hit in a nine inning game.
   v. Calculate the probability that more than three home runs will be hit in a nine inning game.
   vi. The longest baseball game in history was 27 innings. Calculate the probability that there would be 5 home runs hit in a 27 inning game.
   vii. Calculate the expected time in innings between home runs.
   viii. Calculate the expected time in innings until the second home run is hit in a game.
   ix. Calculate the variance of expected time in innings until the second home run is hit in a game.
24. The number of home runs hit in baseball game are distributed as a Poisson distribution with home runs hit at a rate of 3 per nine innings.

Home runs can produce one run, two runs, three runs, or four runs. The number of runs per home run are distributed as follows:

<table>
<thead>
<tr>
<th>Number of Runs per Home Run</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Let S be the total number of runs generated by home runs in a nine inning game.

i. Calculate E(S).
ii. Calculate Var(S).
iii. A season of baseball for a team consists of 162 nine inning games. (Assume there were no extra inning games.) A team agrees to refund all ticket revenue to their fans if the total runs resulting from home runs in the 162 game season are less than R. The team wants to be 99% confident that they will not have to refund the ticket revenue. Using the normal approximation, determine R.

25. The gates for a Purdue football game open 60 minutes prior to the start of the game. Students arriving at a football game is a Poisson process with \( \lambda(t) = 45t - 0.5t^2 \). Time (t) is measured in minutes beginning 90 minutes prior to the start of the game.

Calculate the expected number of students that will be waiting when the gates open.

26. Customers at a styling salon arrive at a rate of \( 3t^2 \) during the morning hours where t is measured in hours from the time the salon opens.

i. Calculate the probability that less than four customers will arrive in the first two hours that the salon is open.
ii. Calculate the probability that less than four customers will arrive during the third hour that the salon is open.
27. Castronova Auto Company classifies drivers as good, average, and bad. Accidents follow a Poisson process for each classification of driver. The good driver has accidents at a rate of 1 per five year period. Average drivers have an accident at a rate of 2.5 per five year period. Bad drivers have an accident at a rate of 6 per five year period.

The distribution of insureds is: Good - 50%; Average – 40%; and Bad – 10%.

i. Calculate the probability that an insured will have zero claims over the next five years.
ii. Calculate the expected claims per insured over the next five years.
iii. Calculate the variance of the expected number of claims over the next five years.

28. Customers at a styling salon arrive at a Poisson rate. The Poisson rate, measured in customers per hour, is constant each day, but varies day to day according to a gamma distribution with a mean of 6 and a variance of 12.

i. Calculate the expected number of customers per hour on any given day.
ii. Calculate the variance of the expected number of customers per hour on any given day.
iii. Calculate the probability that more than four customers will arrive in a one hour period.

29. An archaeologist finds dinosaur bones at a rate of 1 per hour. The bones are randomly distributed as:
   a. Ribs – 50%
   b. Legs – 40%
   c. Skulls – 10%

Calculate the probability that she will find two or more legs in an eight hour day and at least 3 legs in two eight hour days.

30. The availability of a stylist to cut hair at a styling salon follows a Poisson process. From 8 am until noon, a stylist becomes available at a rate of 2 per hour. During the afternoon, a stylist becomes available at a rate of 4 per hour.

Calculate the expected waiting time for a customer that arrives at 11:30 am.
31. The availability of a stylist to cut hair at a styling salon follows a Poisson process. A stylist becomes available at a rate of 4 per hour.

Experienced stylists take 20 minutes to cut hair while new stylists take 30 minutes to cut hair. 25% of the stylists are new stylists.

i. If the only customer waiting takes the first stylist that becomes available, calculate the expected time from arrival until completion of the hair cut.

ii. If the only customer waiting waits until the first experienced stylist becomes available, calculate the expected time from arrival until completion of the hair cut.

32. Renco stock currently sells for 100 per share. Renco does not pay a dividend. A 102-strike one-year European call sells for 8.00. The risk free interest rate is 6% compounded continuously.

Calculate the premium for a 102-strike one-year European put.

33. Tariq LTD stock currently sells for 100 per share. It pays a dividend of 2 every 3 months with the next dividend due in 2 months. The risk free interest rate is 8% compounded continuously. A 104-strike six month European put has a premium of 4.

Calculate the premium for a 104-strike six month European call.

34. Yu Corporation stock currently sells for 25 per share and does not pay a dividend. A 25-strike 3-month European call has a premium of 1.00 while a 25-strike 3 month European put has a premium of 0.75.

Calculate the annual risk free interest compounded continuously.

35. The S&P Index has current price of 1000. The index pays a dividend at a continuous rate of δ. The risk free interest rate is 12% compounded continuously. A one-year European call option with a strike price of 1040 has a premium of 40. A one-year European put option with a strike price of 1040 has a premium of 25.33.

Calculate δ.

36. A bond with semi-annual coupons of 40 sells for 960 immediately after the last coupon. A four year European call option on the bond with a strike price of 1000 has a premium of 20. The annual risk free interest rate is 6% compounded continuously.

Calculate the premium for a four year European put option with a strike price of 1000
37. Renco stock currently sells for 100 per share. Renco does not pay a dividend.

Tariq LTD stock currently sells for 100 per share. It pays a dividend of 2 every 3 months with the next dividend due in 2 months.

A one year European call provides the option of obtaining a share of Renco in exchange for a share of Tariq LTD. The call sells for 15.00.

The risk free interest rate is 5% compounded continuously.

Calculate the premium for a one year European put providing the option of selling a share of Renco in exchange for a share of Tariq LTD.

38. The dollar exchange rate for euros is 0.92 dollars/euro. The dollar interest rate is 6% compounded continuously while the euro interest rate is 5% compounded continuously. The premium for a one year dollar denominated call option with a strike price of K dollars/euro is 0.0928. The premium for a one year dollar denominated put option with a strike price of K dollars/euro is 0.0700.

Calculate K.

39. The dollar exchange rate for euros is 0.92 dollars/euro. The dollar interest rate is 6% compounded continuously while the euro interest rate is 5% compounded continuously. The price of a one year dollar denominated $0.91-strike call is 0.09.

Find the price of the euro denominated one year call option with a strike of 1/0.91.

40. Which of the following are true? For those that are false, explain why:
   i. You can determine the maximum price of an American Put or Call by finding the price of the comparable (same strike price, same termination date, and same underlying asset) European Option.
   ii. The maximum possible price for an American Call option with a strike of 100 on a stock with a current spot price of 90 is 95.
   iii. An American call option on a stock with no dividends should never be exercised.
   iv. An American put option on a stock with a termination date in two years must be worth at least as much as a comparable American Put with a shorter termination date.
   v. A European put option on a stock with a termination date in two years must be worth at least as much as a comparable European Put with a shorter termination date.
41. Renco stock currently sells for 100 per share. Renco does not pay a dividend. A 102-strike one-year European call sells for 8.00. The risk free interest rate is 6% compounded continuously.

Calculate the range of possible premiums for a 102-strike one-year American put.

42. You are given the following call premiums for calls based on Renco Stock with a one year European option.

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>100</th>
<th>105</th>
<th>110</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call Premium</td>
<td>8.00</td>
<td>9.00</td>
<td>3.00</td>
</tr>
</tbody>
</table>

Which of the following are true?

i. No arbitrage is possible.
ii. An arbitrage profit can be made by selling the 100 strike call and buying the 105 strike call.
iii. An arbitrage profit can be made by buying the 100 strike call and selling the 105 strike call.
iv. An arbitrage profit can be made by selling the 105 strike call and buying the 110 strike call.
v. An arbitrage profit can be made by buying the 105 strike call and selling the 110 strike call.

For those where an arbitrage profit can be made, show how.

43. You are given the following put premiums for calls based on Renco Stock with a one year European option.

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>100</th>
<th>110</th>
<th>115</th>
</tr>
</thead>
<tbody>
<tr>
<td>Put Premium</td>
<td>4.06</td>
<td>9.00</td>
<td>13.00</td>
</tr>
</tbody>
</table>

Which of the following are true?

i. No arbitrage is possible.
ii. An arbitrage profit can be made by selling the 100 strike put and buying the 105 strike put.
iii. An arbitrage profit can be made by buying the 100 strike put and selling the 105 strike put.
iv. An arbitrage profit can be made by selling the 105 strike put and buying the 110 strike put.
v. An arbitrage profit can be made by buying the 105 strike put and selling the 110 strike put.

For those where an arbitrage profit can be made, show how.
44. You are given the following put premiums for calls based on Renco Stock with a one year European option.

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>100</th>
<th>105</th>
<th>110</th>
</tr>
</thead>
<tbody>
<tr>
<td>Put Premium</td>
<td>4.06</td>
<td>9.00</td>
<td>13.00</td>
</tr>
</tbody>
</table>

Explain how you would take advantage of the arbitrage using an asymmetric butterfly spread.

45. A six month call option is priced using a period binomial tree. The stock’s current price is 30 and the strike price is 35. The dividend pays a dividend continuously at a rate of 2% per annum and the risk free interest rate is 4% compounded continuously. The volatility of the stock is 30%.

i. Calculate the price of the call option.
ii. Determine the replicating portfolio.

46. A six month put option is priced using a period binomial tree. The stock’s current price is 30 and the strike price is 35. The dividend pays a dividend continuously at a rate of 2% per annum and the risk free interest rate is 4% compounded continuously. The volatility of the stock is 30%.

i. Calculate the price of the put option.
ii. Determine the replicating portfolio.

47. You are given $S_0 = 100$, $K = 95$, $r = 8\%$, $\delta = 0$, $\sigma = 30\%$, $T = 1$, and we use a three period binomial model. Be sure to check for early exercise for the American options.

i. Calculate the price of a European call option.
ii. Calculate the price of an American call option.
iii. Calculate the price of a European put option.
iv. Calculate the price of an American put option.

48. You are given $S_0 = 100$, $K = 95$, $r = 8\%$, $\delta = 8\%$, $\sigma = 30\%$, $T = 1$, and we use a three period binomial model. Be sure to check for early exercise for the American options.

i. Calculate the price of a European call option.
ii. Calculate the price of an American call option.

49. The dollar-ruble exchange rate is 30 dollars per ruble. The continuously compounded risk free interest rate for dollars is 3%. The continuously compounded risk free interest rate for rubles is 8%. The exchange rate volatility is 0.15. A six month dollar denominated European call option to buy rubles has a strike price of 28.

Use a one period binomial tree to price this call option.
50. The underlying asset for a one year European put option is a futures contract on a bushel of corn. The current spot price on corn is 6 per bushel. The put option has a strike price of 6.25 per bushel. The risk free rate is 5% compounded continuously and the volatility is 20%.

Use a one period binomial tree to price this put option.

51. A three period binomial tree is used to price a one year American call option on a non-dividend paying stock with a current spot price of 25. The strike price of option is 24. \( S_{uuu} = 36.9245 \). \( S_{udu} = 30.2312 \).

Calculate the option premium.

52. Renco stock had the following prices at the end of the first 6 weeks of 2010:

<table>
<thead>
<tr>
<th>End of Week</th>
<th>Stock Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>100</td>
</tr>
<tr>
<td>Week 2</td>
<td>105</td>
</tr>
<tr>
<td>Week 3</td>
<td>102</td>
</tr>
<tr>
<td>Week 4</td>
<td>105</td>
</tr>
<tr>
<td>Week 5</td>
<td>98</td>
</tr>
<tr>
<td>Week 6</td>
<td>102</td>
</tr>
</tbody>
</table>

Calculate the historical volatility for Renco stock during this time period.

For problems 53-56, use the tables provided for the MFE exam without interpolation.

53. You are given: \( S_0 = 100 \), \( K = 95 \), \( r = 8\% \), \( \delta = 0 \), \( \sigma = 30\% \), \( T = 1 \).
   
i. Calculate the price of a European call option using the Black Sholes formula.
   
ii. Calculate the price of an American call option.
   
iii. Calculate the price of a European put option using the Black Sholes formula.

54. You are given: \( S_0 = 100 \), \( K = 95 \), \( r = 8\% \), \( \delta = 8\% \), \( \sigma = 30\% \), \( T = 1 \).
   
i. Calculate the price of a European call option using the Black Sholes formula.
   
ii. Calculate the price of European put option using the Black Sholes formula.

55. A six month call option is priced using the Black Sholes formula. The stock’s current price is 30 and the strike price is 35. The dividend pays a dividend continuously at a rate of 2% per annum and the risk free interest rate is 4% compounded continuously. The volatility of the stock is 30%.
   
i. Calculate the price of the call option.
56. A six month put option is priced using the Black Sholes formula. The stock’s current price is 30 and
the strike price is 35. The dividend pays a dividend continuously at a rate of 2% per annum and the
risk free interest rate is 4% compounded continuously. The volatility of the stock is 30%.

i. Calculate the price of the put option.

57. The stock for Cao Chocolate Company has a current spot price of 50. A dividend of 5 will be paid in
3 months. The risk free interest rate is 4% compounded continuously. The volatility of the prepaid
forward price is 30%. Using the Black Sholes formula:

i. Calculate the premium for a six month European call option with a strike of 48.
ii. Calculate the premium for a six month European put option with a strike of 48.

58. The dollar-ruble exchange rate is 30 dollars per ruble. The continuously compounded risk free
interest rate for dollars is 3%. The continuously compounded risk free interest rate for rubles is 8%.
The exchange rate volatility is 0.15. A six month dollar denominated European call option to buy
rubles has a strike price of 28.

i. Use the Garman-Kohlhagen formula to price this call option.
ii. Calculate the premium for a six month dollar denominated European put option to buy
rubles at a strike price of 28.

59. The underlying asset for a one year European call option is a futures contract on a bushel of corn.
The current spot price on corn is 6 per bushel. The call option has a strike price of 6.25 per bushel.
The risk free rate is 5% compounded continuously and the volatility is 20%.

Use the Black formula to calculate the price of this call option.

60. You are given: S_0 = 100, K = 95, r = 8%, δ = 0, σ = 30%, T = 1.

i. Calculate the Delta of a European call option.
ii. Calculate the elasticity (Ω) for the call option.
iii. Calculate the volatility of the call option.
iv. Calculate the Delta of a European put option.
v. Calculate the elasticity (Ω) for the put option.
vi. Calculate the volatility of the put option.
61. You are given: \( S_0 = 100, K = 95, r = 8\%, \delta = 8\%, \sigma = 30\%, T = 1 \).

   i. Calculate the Delta of a European call option.
   ii. Calculate the elasticity (\( \Omega \)) for the call option.
   iii. Calculate the volatility of the call option.
   iv. If the stock is expected to return 16\%, calculate the risk premium on the call option.
   v. If the stock is expected to return 16\%, calculate the Sharpe Ratio for the stock.
   vi. If the stock is expected to return 16\%, calculate the Sharpe Ratio for the call option.
   vii. Calculate the Delta of a European put option.
   viii. Calculate the elasticity (\( \Omega \)) for the put option.
   ix. Calculate the volatility of the put option.
   x. If the stock is expected to return 16\%, calculate the risk premium on the put option.

62. *For a European call option on stock within the Black-Scholes framework, you are given:

   i. The stock price is 85.
   ii. The strike price is 80.
   iii. The call option will expire in one year.
   iv. The continuously compounded risk free interest rate is 5.5\%.
   v. \( \sigma = 0.50 \)
   vi. The stock pays no dividends.

   Calculate the volatility of the call option.

63. The price of a one year European call on Ryan Industries stock is 14.6406. Ryan Industries currently sell for 100. The call has a strike price of 100. The risk free continuously compounded interest rate is 8\%. Ryan Industries pays a dividend at a continuous rate of 8\%.

   Calculate the implied volatility.
64. You are given: \( S_0 = 100, K = 95, r = 8\%, \delta = 0, \sigma = 30\%, T = 1 \). Further you are given that:

<table>
<thead>
<tr>
<th>Call Price</th>
<th>Delta</th>
<th>Gamma</th>
<th>Theta</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.3871</td>
<td>0.7216</td>
<td>0.0112</td>
<td>-0.0256</td>
</tr>
</tbody>
</table>

Assume that a market maker sell 50 calls.

i. Calculate the premium that the market maker will collect for selling the calls.

ii. If the market maker does not hedge, calculate the maximum profit that the market maker can attain.

iii. Determine the range of spot prices at time of termination that will result in this maximum profit.

iv. Determine the market maker’s profit if the spot price at the expiration date is 160.

v. Determine the maximum loss (negative profit) that market maker could incur.

vi. If the market maker delta hedges his position, how many shares of stock should he buy?

vii. What is the cost of the stock bought to delta hedge?

viii. Determine the amount of the loan used to delta hedge.

ix. Assume that the price of the stock changes from 100 to 101 during the first day of after the calls were sold. Calculate the profit (or loss) incurred by the market maker by marking the position to market. The new price of the call is 19.0885 and the new delta is 0.7325.

x. Given the information in ix, what actions should the market maker take to rebalance her delta hedged position.

xi. While ix gives us the exact new price of the call, we could have estimated it using the Greeks. Estimate the price using:
   a. Delta Approximation
   b. Delta-Gamma Approximation
   c. Delta-Gamma-Theta Approximation

xii. Estimate the profit for the first day using the Greeks.

65. You are given the following information for two call options:

<table>
<thead>
<tr>
<th>Greek</th>
<th>Call A</th>
<th>Call B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta</td>
<td>0.6424</td>
<td>0.4935</td>
</tr>
<tr>
<td>Gamma</td>
<td>0.0299</td>
<td>0.0319</td>
</tr>
<tr>
<td>Theta</td>
<td>-0.0106</td>
<td>-0.0102</td>
</tr>
</tbody>
</table>

You have just sold 1000 of Call A.

Determine the what steps you should take to both delta-hedge and gamma-hedge your position.
66. You are given the following information for two call options:

<table>
<thead>
<tr>
<th>Greek</th>
<th>Call A</th>
<th>Call B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta</td>
<td>0.7616</td>
<td>Δ</td>
</tr>
<tr>
<td>Gamma</td>
<td>0.0117</td>
<td>Γ</td>
</tr>
</tbody>
</table>

To create a portfolio that is both delta-hedged and gamma-hedged, you have sold 500 of Call A, purchased 395.27 of Call B and purchased 157.71 shares of the underlying stock.

Determine Δ and Γ.

67. Kicho sells 1000 calls on Renco Stock. Renco stock has a current price of 75 and a volatility of 0.35. Kicho delta hedges his position. Determine the two stock prices on the next day at which Kicho will show a profit of zero.

68. You are given the following for options with the same underlying stock:

<table>
<thead>
<tr>
<th>Option</th>
<th>Call A</th>
<th>Put X</th>
<th>Put Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ</td>
<td>0.70</td>
<td>-0.10</td>
<td>-0.60</td>
</tr>
<tr>
<td>Γ</td>
<td>0.12</td>
<td>0.02</td>
<td>0.04</td>
</tr>
</tbody>
</table>

You have sold 1000 of Call A. You are going to both delta hedge and gamma hedge your position using only positions in Put X and Put Y.

Determine how much of each put you should buy or sell.

69. Liyana sells 100 European calls on Tariq LTD with which expiration date of one year. She wants to delta hedge her position by selling puts on Tariq with the same strike price and same expiry date. You are given that \( r = 0.07 \) and \( \delta = 0.05 \). Furthermore, \( d_1 = 0.52 \) and \( \sigma = 25\% \).

Calculate the number of puts that Liyana should sell to delta hedge.
70. The following are the observed stock prices at the end of the month for the last three months:

<table>
<thead>
<tr>
<th>End of Month</th>
<th>Stock Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>57</td>
</tr>
</tbody>
</table>

i. Calculate the arithmetic average stock price.
ii. Calculate the geometric average stock price.
iii. Calculate the payoff for a 3 month average strike call using the arithmetic average of the stock price at the end of the last three months.

71. The following are the observed stock prices at the end of the month for the last three months:

<table>
<thead>
<tr>
<th>End of Month</th>
<th>Stock Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>X</td>
</tr>
<tr>
<td>3</td>
<td>Y</td>
</tr>
</tbody>
</table>

The arithmetic average stock price is $22 \frac{1}{3}$.

The geometric average stock price is 20.

Calculate the possible values of X.

72. You are given:

$S = 75, K = 70, r = 0.10, \delta = 0.04, \sigma = 0.40, T = 9$ months

An Asian Option uses the average prices at the end of 3 months, 6 months, and 9 months.

Use a 3 period binomial tree to calculate the price of:

i. An arithmetic average price put option
ii. A geometric average strike call option
iii. A rebate option that pays a rebate of 10 at expiration if the price of the stock ever reaches 80.
iv. A gap call option with a trigger of 80 and a strike price of 70.
v. A gap put option with a trigger of 50 and a strike price of 40.
73. You are given:

\[ S = 75, \ K = 80, \ r = 0.10, \ \delta = 0, \ \sigma = 0.40, \ T = 9 \text{ months} \]

i. Calculate the price of a standard call.

ii. Determine the price of a knock out call option with a barrier of 78.

iii. Determine the price of a knock in call option with a barrier of 78.

74. You are given:

\[ S = 75, \ K = 70, \ r = 0.10, \ \delta = 0.04, \ \sigma = 0.40 \]

You purchase a CallonCall option that gives you the right to purchase a 6 month call option at the end of six months at a price of $2.50.

Use a spreadsheet to calculate the minimum stock price at the end of six months such that you would exercise the option.

75. You are given:

\[ S = 75, \ K = 80, \ r = 0.10, \ \delta = 0, \ \sigma = 0.40 \]

You purchase a CallonCall option that gives you the right to purchase a 6 month call option at the end of three months at a price of $9.00 for a premium of 2.25.

Calculate the premium for an equivalent PutonCall.

76. You are given:

\[ S = 75, \ r = 0.10, \ \delta = 0.04, \ \sigma = 0.40 \]

You purchase a one year Gap Call option that requires you to exercise the option if the price of the stock is greater than 70. The strike price is 80.

Calculate the premium for this option using Black-Sholes.
77. You purchase an exchange option that gives you the right to receive Google stock in exchange for Microsoft stock at the end of one year at an exchange rate equal to the current stock price.

You are also given:

<table>
<thead>
<tr>
<th>Stock</th>
<th>$S_0$</th>
<th>$\delta$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Google</td>
<td>570</td>
<td>0</td>
<td>25%</td>
</tr>
<tr>
<td>Microsoft</td>
<td>30</td>
<td>2%</td>
<td>36%</td>
</tr>
</tbody>
</table>

You are also given the correlation between the returns is 80%.

Calculate the price of this exchange option using Black-Sholes.

78. You are given:

$S = 50$, $\alpha = 0.10$, $\delta = 0.04$, $\sigma = 0.40$

Determine:

i. Expected Stock Price in 18 months
ii. Median Stock Price in 18 months
iii. The probability that the stock price will be less than 40 in 18 months.
iv. The probability that the stock price will be more than 60 in 18 months.
v. The 90% confidence level for the stock price at the end of 18 months.
vi. Calculate the 95% confidence interval for the 18 month continuously compounded return.
vii. The expected stock price assuming the stock price exceeds 60 at the end of 18 months.
viii. The expected payoff on a 18 month European call option with a strike price of 40.

79. Renco stock had the following prices at the end of the first 6 weeks of 2010:

<table>
<thead>
<tr>
<th>End of</th>
<th>Stock Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>100</td>
</tr>
<tr>
<td>Week 2</td>
<td>105</td>
</tr>
<tr>
<td>Week 3</td>
<td>102</td>
</tr>
<tr>
<td>Week 4</td>
<td>105</td>
</tr>
<tr>
<td>Week 5</td>
<td>98</td>
</tr>
<tr>
<td>Week 6</td>
<td>102</td>
</tr>
</tbody>
</table>

Renco does not pay a dividend. Use this information to estimate $\alpha$ and $\sigma$. 
80. The following twelve random numbers were generated from a uniform distribution from zero to one.

0.2, 0.7, 0.5, 0.6, 0.3, 0.6, 0.5, 0.4, 0.3, 0.8, 0.5, 0.5

Calculate $Z$.

81. The following four random numbers were generated from a uniform distribution from zero to one.

0.8531
0.3264
0.6879
0.0055

Using the inversion method, determine the standard normal variable $z$.

82. You are given:

   i. Stock prices follow a lognormal model
   ii. $S_0 = 50$, $\alpha = 0.10$, $\delta = 0.04$, $\sigma = 0.40$
   iii. The following random numbers drawn from a uniform distribution from zero to one - 0.4761, 0.5557, 0.6141, 0.4090, and 0.4522

The stock price at the end of one year is simulated using the inversion method and one random number per trial to project the one year stock price.

Based on these five trials, calculate:

   i. The expected stock price at the end of one year.
   ii. The expected payoff on a one year European call option with a strike price of 53.
   iii. The probability that a one year European put option with a strike price of 50 will have a positive payoff.
83. You are given:

i. Stock prices follow a lognormal model
ii. $S_0 = 50, r = 0.06, \delta = 0.01, \sigma = 0.20$
iii. The following random numbers drawn from a uniform distribution from zero to one - 0.3300, 0.5557, 0.4090, and 0.6141.

The stock price at the end of one year is simulated using the inversion method. The stock price is simulated using one random number per quarter to project the one year stock price.

Calculate the price of a one year American put option with a strike price of 50.

Develop a spreadsheet that calculates 100 trials at a time. Calculate the average price of the put using these 100 trials. Record your price. Hit F9 to recalculate your spreadsheet. Record the average price after the recalculation. Record a total of 10 trials of 100. Record the average price using these 10 trials of 100. Email spreadsheet to Nick.

84. Estimate the standard deviation of the sample mean given 1 million trials based on a 5 simulations that produce the following prices for a put: 2, 4, 6, 3, 7

85. You are given:

i. Stock prices follow a lognormal model
ii. $S_0 = 50, r = 0.06, \delta = 0.04, \sigma = 0.40$
iii. The following random numbers drawn from a standard normal distribution - 0.6, -1.2, 0.2, 0.3, 1.5, and -0.1

The stock price at the end of six months is simulated using the inversion method. The stock price is simulated using one random number per month to project the six month stock price.

Calculate the simulated present value of a 6 month Asian arithmetic average strike call option.
86. You are given:

i. Stock prices follow a lognormal model
ii. \( S_0 = 50, r = 0.06, \delta = 0.04, \sigma = 0.40 \)
iii. The following random numbers drawn from a standard normal distribution -
    0.6, -1.2, 0.2, 0.3, 1.5, and -0.1

The stock price at the end of two months is simulated using the inversion method. The stock price is simulated using one random number per month to project the two month stock price.

i. Simulate the stock price at the end of one month and at the end of two months for each set of random numbers. You will have three sets of stock prices as there are six random numbers.
ii. For each of the three sets, calculate the present value of the payoff for a two month European call option with strike price of 51.
iii. For each of the three sets, calculate the present value of the payoff for a two month American call option with strike price of 51.
iv. Calculate the price of a two month European call option with a strike price of 51 using Black-Sholes.
v. For each of the three sets, simulate the present value of the payoff using the Control Variate Method.
vi. Use this sample of three to calculate \( \beta \) under the Boyle modification to the Control Variate Method.

87. The price of a stock is assumed to follow Brownian motion. The price of the stock on Day 2 is 48. What is the probability that the price of the stock on Day 8 is less than 50.

88. A stock price follows arithmetic Brownian motion with drift of 0.25 and volatility of 0.40. The stock price at Day 0 was 60 and the stock price at Day 2 was 65. Calculate the probability that the stock price at Day 6 will be between 65 and 68.

89. \( S(t) \) is the stock price at time \( t \) and is modeled using geometric Brownian motion. The continuously compounded expected rate of return is 12%. The stock does not pay dividends. The stock volatility is 25%. The continuously compounded risk free interest rate is 4%. The current stock price is 40.

Calculate the probability that the return on the stock after 2 years will be greater than risk free return.
90. \( S(t) \) follows an Itô Process of the form

\[
dS(t) = 0.10S(t)dt + 0.15S(t)dZ(t)
\]

You are given that the \( S(1) = 25 \) and \( S(3) = 30 \).

Find the probability that \( S(10) < 50 \)

91. \( S(t) \) follows an Itô Process of the form

\[
dS(t) = 0.10S(t)dt + 0.15S(t)dZ(t)
\]

The stock does not pay a dividend and has a current price of 48. The risk premium on the stock is 4%.

Calculate the premium for a 3 year European put option with a strike of 55.

92. You are given \( S(t) = \alpha t + \sigma(t) \).

Use Itô’s lemma to determine \( dS(t) \).

93. The price of a stock at time \( t \) is \( S(t) \). The stock does not pay dividends. \( S(t) \) follows arithmetic Brownian motion with \( \alpha = 0.5 \) and \( \sigma = 0.40 \).

The price at time \( t \) of a derivative based on the stock is \( D(t) \).

You are given:

\[
\frac{\partial D}{\partial S} = 0.5 \quad \frac{\partial^2 D}{\partial S^2} = 0.05 \quad \frac{\partial D}{\partial t} = 0.1
\]

Determine the drift and noise terms.

94. A stock that pays no dividends is modeled using a mean reversion model with parameters \( \alpha = 50 \), \( \lambda = 0.5 \), and \( \sigma = 0.2 \). Determine the drift and noise terms of a process following \( S(t)^2 - t \).

95. For a non-dividend paying stock, you are given:

i. \( d[\ln S(t)] = 0.08dt + 0.4dZ(t) \)

ii. The Sharpe ratio of a call on this asset is 0.3375

Calculate the continuously compounded risk free interest rate.
96. The following three processes are given for non-dividend paying assets. Determine A for asset W.

\[ d[\ln W(t)] = Adt + 0.15dZ(t) \]
\[ d[\ln X(t)] = 0.12dt + 0.2dZ(t) \]
\[ d[\ln Y(t)] = 0.13dt + 0.1dZ(t) \]

97. For the following two assets, we are given:

i. Asset W does not pay dividends;
ii. The risk free rate is .05.
iii. \( dW(t) = 0.12W(t)dt + 0.2W(t)dZ(t) \)
iv. \( dX(t) = 0.07X(t)dt + 0.1X(t)dZ(t) \)

Determine the dividend rate on Asset X.

98. A 4-month European call option on a stock has strike price 50. Using the following information on the underlying asset, find the price of the option.

The stock follows the Ito process:
\[ dS(t) = 0.14S(t)dt + 0.4S(t)dZ(t) \]
The current stock price is 50.
The stock pays out dividends proportional to its price at a rate of 0.05.
The Sharpe ratio is 0.27.

99. You are given:

i. \( S(0) = 13 \)
ii. The continuously compounded rate of return on the stock is 15%.
iii. The continuously compounded risk free interest rate is 6%.
iv. The volatility of the stock is 25%.
v. The stock does not pay dividends.

You have a claim that pays \( S(5)^3 \) at time 5.

Calculate the value of this claim at time 0.
100. You are given:

i. \( S(0) = 13 \)
ii. \( S(2) = 20 \)
iii. The continuously compounded rate of return on the stock is 15%.
iv. The continuously compounded risk free interest rate is 6%.
v. The volatility of the stock is 25%.
vi. The stock does not pay dividends.

You have a claim that pays \( S(5)^3 \) at time 5.

Calculate the expected value of this claim.

101. \( S(t) \) is the value of a stock. You are given:

i. \( \alpha = 0.12 \)
ii. \( r = 0.05 \)
iii. \( \delta = 0.04 \)
iv. \( \sigma = 0.20 \)
v. \( S(0) = 20 \)
vi. \( E[S(1)^4] = 40 \)

Determine possible values of \( a \).

102. Given that a stock has \( \alpha=0.14, \delta=0.05, r=0.05, \sigma=0.3 \), find:

\[ F_{2,6}[S(6)^3]/S(2)^3 \]

103. You are given the following:

<table>
<thead>
<tr>
<th>Years</th>
<th>Zero Coupon Bond Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.968523</td>
</tr>
<tr>
<td>2</td>
<td>0.929017</td>
</tr>
<tr>
<td>3</td>
<td>0.883887</td>
</tr>
<tr>
<td>4</td>
<td>0.835359</td>
</tr>
<tr>
<td>5</td>
<td>0.783526</td>
</tr>
</tbody>
</table>

i. Calculate the annual effective spot rates for 1 year through 5 years.
ii. Calculate \( r_0(1,2), r_0(2,3), r_0(3,4), \) and \( r_0(4,5) \)
iii. Calculate \( r_0(2,5) \)
iv. Calculate \( P(2,5) \)
v. Calculate \( P(3,5) \)
vi. Calculate \( P(1,4) \)
104. You are given the following forward rates expressed at continuously compounded interest rates:

<table>
<thead>
<tr>
<th>Period</th>
<th>Forward Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0(0,1)$</td>
<td>0.02</td>
</tr>
<tr>
<td>$r_0(1,2)$</td>
<td>0.03</td>
</tr>
<tr>
<td>$r_0(2,3)$</td>
<td>0.04</td>
</tr>
<tr>
<td>$r_0(3,4)$</td>
<td>0.05</td>
</tr>
</tbody>
</table>

i. Calculate $P(0,4)$
ii. Calculate $P(2,4)$
iii. Calculate $r(0,4)$

105. You are given the following interest rate tree:

The probability of an up or down movement in the interest rates is 50% at any time. The interest rates in the tree are one year continuously compounded forward rates.

i. Calculate $P(0,1)$
ii. Calculate $P(0,2)$
iii. Calculate $P(0,3)$
iv. Calculate $P(1,3)$
v. Calculate the premium for a 2 year call on a one year bond with a strike price of .9
vi. Calculate the premium for a 2 year put on a one year bond with a strike price of .9

106. *You are given the following Black-Derman-Toy tree for effective annual interest rates:

Calculate $R$. 

\[
\begin{align*}
0.172 \\
0.126 \\
0.090 & \quad 0.135 \\
0.093 & \quad R
\end{align*}
\]
107. *You are given the following Black-Derman-Toy tree for effective annual interest rates:

0.172
0.126
0.090
0.093
0.106

i. Calculate the P(0,3).
ii. Calculate the price of a caplet which guarantees the interest rate during the third year will not exceed 12% on an amount of 100.
iii. Determine the volatility in one year on a two year bond.

108. You are given the following information from the market.

<table>
<thead>
<tr>
<th>Maturity Years</th>
<th>Bond Price</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.943396</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.873439</td>
<td>0.07</td>
</tr>
</tbody>
</table>

The following is a market consistent Black-Derman-Toy interest tree for annual effective interest rates:

\[
\begin{align*}
  r_u \\
  r_0 \\
  r_d
\end{align*}
\]

Calculate \( r_0 \), \( r_u \) and \( r_d \)

109. A bond interest rate is modeled using Rendelman-Bartter. You are given:

i. \( a = .001 \)
ii. \( \sigma = 0.08 \)
iii. Current bond interest rate is 5%.

Calculate the probability that the bond interest rate will be more than 5% in two years.
110. You are given:
   a. $\sigma = 0.07$
   b. $P(0,1) = 0.943396$
   c. $P(0,2) = 0.873439$
   d. Use Black Formula to calculate option prices.

   i. Calculate the premium for a one year call on a one year bond with a strike of 0.90.
   ii. Calculate the premium for a one year put on a one year bond with a strike of 0.90.

111.
Answers

1.
   i. 0.16
   ii. $e^{-0.16t}$
   iii. $(0.05)e^{-0.16t}$ for $j = 1; (0.01)e^{-0.16t}$ for $j = 2; (0.10)e^{-0.16t}$ for $j = 3$
   iv. $\frac{5}{16}(1 - e^{-0.16t})$ for $j = 1; \frac{1}{16}(1 - e^{-0.16t})$ for $j = 2; \frac{10}{16}(1 - e^{-0.16t})$ for $j = 3$
   v. $(1 - e^{-0.16t})$
   vi. $5/16$ for $j = 1; 1/16$ for $j = 2; 10/16$ for $j = 3$
   vii. 0.092410
   viii. 0.092410
   ix. 0.171157
   x. 0.067104
   xi. $e^{-0.05t}$ for $j = 1; e^{-0.01t}$ for $j = 2; e^{-0.10t}$ for $j = 3$
   xii. $1 - e^{-0.05t}$ for $j = 1; 1 - e^{-0.01t}$ for $j = 2; 1 - e^{-0.10t}$ for $j = 3$

2.
   i. $\frac{1}{50-t}$ for $j = 1; \frac{1}{20-t}$ for $j = 2$
   ii. $\frac{1}{70-2t}$
   (50-t)(20-t)
   iii. $\frac{50-t}{50}$ for $j = 1; \frac{20-t}{20}$ for $j = 2$
   iv. $\frac{1000}{(50-t)(20-t)}$
   v. $\frac{(20-t)}{1000}$ for $j = 1; \frac{(50-t)}{1000}$ for $j = 2$
   vi. $\frac{(40t-t^2)}{2000}$ for $j = 1; \frac{(100t-t^2)}{2000}$ for $j = 2$
   vii. $\frac{1000-(50-t)(20-t)}{1000}$
   $= \frac{70t-t^2}{1000}$
   viii. $\frac{t}{50}$ for $j = 1; \frac{t}{20}$ for $j = 2$
   ix. 0.0195
   x. 0.02
   xi. 0.019871
   xii. 0.020408
   xiii. 0.098
   xiv. 0.10
   xv. 0.0475

3. No answer is provided
4.
  i. 0.766271
  ii. 0.129314
  iii. 55.330444
  iv. 0.012709
  v. 0.439965

5. 0.180167
6. 0.180520
7. 0.085951
8. 0.0704
9. 802.56
10. 0.35
11. 25,000
12. 63.64

13.
  i. 44,438.16
  ii. 48,437.44
  iii. 3999.28
  iv. 270,779.80
  v. -17,246.96
  vi. 253,532.84
  vii. 3250.89
  viii. 30,517.00

14.
  i. 231.01
  ii. |  |  |  |
      t | \( tV_x \) | \( tV_x^e \) | \( tAS \) |
      0 | 0         | 0          | 0       |
      1 | 47.50     | -88.71     | -41.22  |
      2 | 49.05     | -46.39     | 2.65    |
      3 | 0         | 0          | 0       |

15. 0, -51.65, -9.77, -14.43
16. 0.02443
17. \( g = 0.217 \) and \( h = 516.89 \)
18. 15.39%
19.
  i. 621.1
  ii. 235.33479
<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20.</td>
<td></td>
<td>i.</td>
<td>3599.51</td>
<td></td>
<td>ii.</td>
<td>1484.79</td>
<td></td>
<td>iii.</td>
<td>928.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>iv.</td>
<td>5823.44</td>
<td></td>
<td>v.</td>
<td>240.46</td>
<td></td>
</tr>
<tr>
<td>21.</td>
<td></td>
<td>i.</td>
<td>128,854.27</td>
<td></td>
<td>ii.</td>
<td>53,152.09</td>
<td></td>
<td>iii.</td>
<td>-9,280.81</td>
</tr>
<tr>
<td>22.</td>
<td>35.125%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23.</td>
<td></td>
<td>i.</td>
<td>3</td>
<td></td>
<td>ii.</td>
<td>3</td>
<td></td>
<td>iii.</td>
<td>0.049787</td>
</tr>
<tr>
<td></td>
<td></td>
<td>iv.</td>
<td>0.100819</td>
<td></td>
<td>v.</td>
<td>0.352768</td>
<td></td>
<td>vi.</td>
<td>0.060727</td>
</tr>
<tr>
<td></td>
<td></td>
<td>vii.</td>
<td>3 innings</td>
<td></td>
<td>viii.</td>
<td>6 innings</td>
<td></td>
<td>ix.</td>
<td>2 innings</td>
</tr>
<tr>
<td>24.</td>
<td></td>
<td>i.</td>
<td>6.3</td>
<td></td>
<td>ii.</td>
<td>15.9</td>
<td></td>
<td>iii.</td>
<td>902.55</td>
</tr>
<tr>
<td>25.</td>
<td>15,750</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26.</td>
<td></td>
<td>i.</td>
<td>0.042380</td>
<td></td>
<td>ii.</td>
<td>0.000007528</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27.</td>
<td></td>
<td>i.</td>
<td>0.217022</td>
<td></td>
<td>ii.</td>
<td>2.1</td>
<td></td>
<td>iii.</td>
<td>4.29</td>
</tr>
<tr>
<td>28.</td>
<td></td>
<td>i.</td>
<td>6</td>
<td></td>
<td>ii.</td>
<td>18</td>
<td></td>
<td>iii.</td>
<td>0.570645</td>
</tr>
<tr>
<td>29.</td>
<td>0.820292</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.</td>
<td>24.48</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
31.  
   i. 37.5 minutes  
   ii. 40 minutes  

32. 4.06  
33. 0.17  
34. 4.02\%  
35. 6.50\%  
36. 126.88  
37. 7.21  
38. 0.90504  
39. 0.08585  

40.  
   i. False  
   ii. False  
   iii. True  
   iv. True  
   v. False  

41. (4.06, 102)  

42.  
   i. False  
   ii. False  
   iii. True  
   iv. True  
   v. False  

43.  
   i. True  
   ii. False  
   iii. False  
   iv. False  
   v. False  

44. Buy 5 100 Strike Options  
   Sell 10 105 Strike Options  
   Buy 5 110 Strike Options  
   Other answers are possible but the ratio must remain 1:2:1  

45.  
   i. 1.07916  
   ii. Buy 0.188195 share of stock and borrow 4.56668  

46.  
   i. 5.685  
   ii. Sell 0.801855 shares and lend 29.7403
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>47.</td>
<td>i. 18.28255</td>
<td>ii. 18.28255</td>
<td>iii. 5.9786</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>iv. 6.6779</td>
</tr>
<tr>
<td>48.</td>
<td>i. 13.9415</td>
<td>ii. 14.1830</td>
<td></td>
</tr>
<tr>
<td>49.</td>
<td>2.1146</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.</td>
<td>0.6996</td>
<td></td>
<td></td>
</tr>
<tr>
<td>51.</td>
<td>3.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>52.</td>
<td>0.3667</td>
<td></td>
<td></td>
</tr>
<tr>
<td>53.</td>
<td>i. 18.3858</td>
<td>ii. 18.3858</td>
<td>iii. 6.0819</td>
</tr>
<tr>
<td>54.</td>
<td>i. 13.1913</td>
<td>ii. 8.5758</td>
<td></td>
</tr>
<tr>
<td>55.</td>
<td>1.0750</td>
<td></td>
<td></td>
</tr>
<tr>
<td>56.</td>
<td>5.6805</td>
<td></td>
<td></td>
</tr>
<tr>
<td>57.</td>
<td>i. 2.9331</td>
<td>ii. 4.9328</td>
<td></td>
</tr>
<tr>
<td>58.</td>
<td>i. 1.9534</td>
<td>ii. 0.7129</td>
<td></td>
</tr>
<tr>
<td>59.</td>
<td>0.5033</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60.</td>
<td>i. 0.7224</td>
<td>ii. 3.9291</td>
<td>iii. 1.1787</td>
</tr>
<tr>
<td>61.</td>
<td>i. 0.5774</td>
<td>ii. 4.3771</td>
<td>iii. 1.3131</td>
</tr>
</tbody>
</table>
vii. – 0.3457
viii. = 4.0311
ix. 1.2093
x. -0.3225
62. 1.39
63. 0.40
64.
   i. 919.355
   ii. 995.925
   iii. Range of 0 to 95
   iv. -2254.075
   v. Infinity
   vi. 36.08
   vii. 3608
   viii. 2688.645
   ix. 0.42 Profit
   x. Borrow 55.045 to buy 0.545 additional shares at a price of 101 per share.
   xi.
      a. 19.1087
      b. 19.1143
      c. 19.0887
   xii. 0.41
65. Buy 937.3 of Call B and buy 179.84 shares of stock
66. $\Delta = 0.5644$
    $\Gamma = 0.0148$
67. 73.63 or 76.37
68. Buy 12,500 Put X
    Sell 3250 Put Y
69. 231.66
70.
   i. 50.000
   ii. 49.748
   iii. 7.000
71. 25 and 32
72.
   i. 4.3047
   ii. 5.6696
   iii. 1.8801
   iv. 8.8432
   v. -0.4712
73.
   i. 10.4718
   ii. 0
   iii. 10.4718
74. 56.00
75. 0.5560
76. 10.6798
77. 59.76
78.
   i. 54.7087
   ii. 48.5226
   iii. 0.3483
   iv. 0.3336
   v. (21.6746, 108.6254)
   vi. (-0.9902, .9302)
   vii. 85.9170
   viii. 28.05
79. α = 0.2733 and σ = 0.3667
80. -0.1
81. 1.05, -0.45, 0.49, -2.54
82.
   i. 49.227
   ii. 0.4076
   iii. 3/5
83. 1.765
84. 0.002074
85. 4.7868
86.
   i. Simulation 1 => 53.31966 => 46.18897
      Simulation 2 => 50.91293 => 52.44463
      Simulation 3 => 59.15896 => 58.18811
   ii. 0, 1.4303, 7.1166
   iii. 2.3081, 1.4303, 8.1183
   iv. 2.7954
   v. 5.1035, 2.7954, 3.7971
   vi. 0.9188
87. 0.7939
88. 0.8882
89. 0.5199
90. 0.3897
91. 3.894
92. 2tαdt + σdZ
93. Drift = 0.354 and Noise = 0.2
94. \[50S(t) - S(t)^2 - 0.96]dt + 0.4S(t)dZ(t)
95. 0.025
96. 0.12625
97. 0.015
98. 4.7457
99. 10,222.52
100. 54,159.94
101. -153.984 or 1.1978
102. 2.9447
103.
\[\begin{align*}
i. & \quad 0.0325, 0.0375, 0.0420, 0.0460, \text{ and } 0.0500 \\
ii. & \quad 0.042525, 0.051059, 0.058092, \text{ and } 0.066154 \\
iii. & \quad 0.058417 \\
iv. & \quad 0.84339 \\
v. & \quad 0.88645 \\
vi. & \quad 0.86251
\end{align*}\]
104.
\[\begin{align*}
i. & \quad 0.869358 \\
ii. & \quad 0.913931 \\
iii. & \quad 0.035
\end{align*}\]
105.
\[\begin{align*}
i. & \quad 0.923116 \\
ii. & \quad 0.852314 \\
iii. & \quad 0.787414 \\
iv. & \quad 0.852996 \\
v. & \quad 0.023063 \\
vi. & \quad 0.002731
\end{align*}\]
106. 0.106
107.
\[\begin{align*}
i. & \quad 0.727979 \\
ii. & \quad 1.45028 \\
iii. & \quad 0.1518
\end{align*}\]
108. \(r_0 = 0.06, r_d = 0.074523, r_u = 0.085722\)
109. 0.4671
110.
\[\begin{align*}
i. & \quad 0.038157 \\
ii. & \quad 0.013774
\end{align*}\]