1. Lisa wants to have 1,000,000 on her 40\textsuperscript{th} birthday. Lisa’s 20\textsuperscript{th} birthday is today.

Lisa wants to invests $K$ in an account which has a force of interest of $\delta_t = 0.04 + 0.0005t^2$ so that she will exactly achieve her goal on her 40\textsuperscript{th} birthday.

Determine $K$.

Solution:

\[(PV)a(20) = 1,000,000\]

\[
(PV)e^{\int_0^{20} \delta_t dt} = 1,000,000 \Rightarrow (PV)e^{\int_0^{20} (0.04 + 0.0005t^2) dt} = 1,000,000
\]

\[
=> (PV)e^{0.04 \times \frac{0.0005(20)^3}{3}} = 1,000,000 \Rightarrow PV = \frac{1,000,000}{e^{2.13333333}} = 118,441.83
\]
2. Grant invests 10,000 in an account earning simple interest of 10% per year.

Simon invests 10,000 in an account earning compound interest.

During the 10th year, Grant and Simon earn the same annual effective interest rate.

How much money does Simon have at the end of 20 years?

Solution:

\[
Grant \implies i_n = \frac{s}{1 + s(n-1)} \implies i_{10} = \frac{0.10}{1 + (0.10)(10-1)} = 0.052631579
\]

\[
Simon \implies i_n = i \implies i_{10} = 0.052631579 \implies i = 0.052631579
\]

Simon has \((10,000)(1.052631579)^{20} = 27,895.10\)
3. You are given that \( d^{(2)} = 0.06 \).

Calculate the equivalent \( i^{(12)} \).

**Solution:**

\[
\left( 1 + \frac{i^{(12)}}{12} \right)^{12} = \left( 1 - \frac{d^{(2)}}{2} \right)^{-2} = \left( 1 - \frac{0.06}{2} \right)^{-2} = (0.97)^{-2}
\]

\[
\left( 1 + \frac{i^{(12)}}{12} \right) = (0.97)^{-\frac{2}{12}}
\]

\[
i^{(12)} = \left[ (0.97)^{-\frac{2}{12}} - 1 \right] (12) = 6.10733\%
\]