1. Hannah is the beneficiary of a trust that will pay her an annual payment of 10,000 with the first payment made twelve years from today. Once the payments beginning they will be made forever to Hannah or her descendants.

Using an interest rate of 6.89%, calculate the present value of Hannah’s payments.

**Solution:**

\[
PV = 10,000 a_{\overline{\infty} | 11} = \frac{10,000}{0.0689} \left( \frac{1}{1.0689} \right)^{11} = 69,738.55
\]
2. Sarah is the beneficiary of the Kinney Scholarship. As such, she will receive payments at the beginning of each month for the next five years. The first payment is 1000. The second payment is 1000(1.02). The third payment is 1000(1.02)^2. Each subsequent payment is 1.02 times the previous payment.

Calculate the present value of these payments at 6% compounded monthly.

**Solution:**

Time 0 = 1000

Time 1 = 1000(1.02)v

Time 2 = 1000(1.02)^2v^2

...Time 59=1000(1.02)^{59}v^{59}

\[ i^{(12)} \frac{0.06}{12} = 0.005 \]

\[ PV = \frac{1000 - 1000(1.02)^{60}v^{60}}{1 - \frac{1.02}{1.005}} = \frac{1000 - 2432.465002}{-0.014925373} = 95,975.16 \]
3. Yi bought a new car for 27,000 using a loan from Bian Bank. The loan is being repaid with 24 monthly payments at a nominal annual interest rate of 9% compounded monthly.

Yi makes the first nine payments. She forgets to make the payment at the end of the 10\textsuperscript{th} month. She then makes the next seven payments as scheduled. Right after making the payment at the end of the 17\textsuperscript{th} month, Yi decides to sell her car and she will need to repay the outstanding loan balance at that time.

Determine the outstanding loan balance that Yi will need to pay to Bian Bank.

**Solution:**

First, determine the amount of the payment:

\[ N=24, \ i/Y=9/12=.75, \ PV=-27000, \ FV=0, \ CPT \ PMT=1233.488042 \]

Compute OLB retrospectively, and then add back in the forgotten payment, accumulated to time 17

\[ OLB_{17} = 27000(1+i)^{17} - PS_{17} + P(1+i)^7 \]

\[ S_{17} = \frac{(1.0075)^{17} - 1}{0.0075} = 18.05927394 \]

\[ P = 1233.488042 \]

\[ OLB_{17} = 27000(1.0075)^{17} - 1233.488042(18.05927394) + 1233.488042(1.0075)^7 \]

\[ = 9680.826093 \]
4. Lisa has won the lottery! She has the following two options to receive her winnings:

   a. An annuity due with quarterly payments of 100,000 for 20 years; or
   b. A perpetuity immediate with annual payments of $P$.

Using a nominal annual interest rate of 8% compounded quarterly, these two options have the same present value.

Calculate $P$.

**Solution:**

\[
\frac{i^{(4)}}{4} = \frac{0.08}{4} = 0.02
\]

\[
(1 + i) = (1 + 0.02)^4 = 1.08243216 \implies i = 0.08243216
\]

\[
100,000a_{\overline{80}|0.02} = Pa_{\overline{\infty}|0.08243216}
\]

\[
100,000\left(\frac{1 - 1.02^{-80}}{0.02}\right)(1.02) = \frac{P}{0.08243216}
\]

\[
4,053,940.386 = \frac{P}{0.08243216}
\]

\[
P = 334,175.0626
\]
5. Nick is 20 years old today. Nick wants to have $1,000,000 on his 65th birthday. Nick will invest an amount of Q today and on each future birthday with a final investment on his 64th birthday.

The payments of Q will be invested into the Carmer Fund which pays an annual effective interest rate of 9.5%.

Calculate the Q that will result in Nick having $1,000,000 on his 65th birthday.

**Solution:**

Calculator set to BGN:

N=45, I/Y=9.5, PV=0, FV=-1,000,000, CPT PMT=Q=1486.11
6. Li Corporation is building a new plant. The expected cash flows from the plant are:

<table>
<thead>
<tr>
<th>Time</th>
<th>Cash Flow in Millions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-100</td>
</tr>
<tr>
<td>1</td>
<td>+5</td>
</tr>
<tr>
<td>2</td>
<td>X</td>
</tr>
<tr>
<td>3</td>
<td>+70</td>
</tr>
<tr>
<td>4</td>
<td>+30</td>
</tr>
</tbody>
</table>

The internal rate of return for this set of cash flows is 9%.

Calculate the Net Present Value at 7%.

**Solution:**

\[
0 = -100 + \frac{5}{1.09} + \frac{X}{1.09^2} + \frac{70}{1.09^3} + \frac{30}{1.09^4}
\]

\[
20.1072441 = \frac{X}{1.09^2} \implies X = 23.88941672
\]

\[
CF_0 = -100
\]

\[
C01 = 5
\]

\[
F01 = 1
\]

\[
C02 = 23.88941672
\]

\[
F02 = 1
\]

\[
C03 = 70
\]

\[
F03 = 1
\]

\[
C04 = 30
\]

\[
F04 = 1
\]

\[
NPV, i = 7, CPT = 5.566546699 \text{ million}
\]
7. Daiana invested 10,000 into the Houser Fund two years ago. One year ago, the amount in the fund had decreased to 8000. Daiana, being an optimist, decided to invest an additional amount of $Y$ into the Fund at that time. Today, Daiana has 21,175.

Daiana’s annual effective time weighted return over the two year period was 10%.

Determine $Y$.

**Solution:**

$B_0 = 10,000$

$B_1 = 8,000$

$C_1 = Y$

$B_2 = 21,175$

\[
1 + j_1 = \frac{8000}{10,000}
\]

\[
1 + j_2 = \frac{21,175}{8000 + Y}
\]

\[
(1.10) = \left[ \frac{8000}{10000} \cdot \frac{21175}{8000 + Y} \right]^{1/2}
\]

\[
1.10^2 = (0.8) \cdot \left( \frac{21175}{8000 + Y} \right)
\]

\[
(1.5125)(8000 + Y) = 21,175 \implies Y = 6000
\]
8. Kristen borrows 42,000 at an annual effective interest rate of 12.3%. She agrees to repay the loan with three payments. The first payment is 22,000 at the end of 2 years. The second payment is 20,000 at the end of 4 years. The last payment is at the end of 6 years.

Determine the last payment.

**Solution:**

\[
42,000 = 22,000(1.123)^{-2} + 20,000(1.123)^{-4} + P(1.123)^{-6}
\]

\[
11,980.22734 = P(1.123)^{-6}
\]

\[
P = 24,029.44
\]
9. Tony loans 8000 to his friend Jiayi. Jiayi agrees to repay the loan with payments of 5000 at the end of one year and 5000 at the end of four years. Tony reinvests the payments at an annual effective interest rate of $r\%$.

Taking into account reinvestment, Tony will realize an annual effective return of 8%.

Determine $r$.

**Solution:**

\[
8000(1.08)^1 = 5000(1 + r)^3 + 5000
\]

\[
5883.91168 = 5000(1 + r)^3
\]

\[
r = 0.055760429
\]
10. Millie invests 1000 in an account that earns simple interest at an annual rate of \( s \).

Tumi invests 1000 in an account that earns compound interest at an annual rate of 8%.

During the 10\(^{th}\) year, Millie and Tumi earn the same amount of interest.

Determine how much money Millie has at the end of the 10 years.

Solution:

Amount of interest in 10th year is \( A(10) - A(9) \)

Tumi Year 10: \( 1000(1.08)^{10} - 1000(1.08)^9 = 159.9203702 \)

Millie Year 10: \( 1000(1 + 10s) - 1000(1 + 9s) = 1000s = 159.9203702 \)

\( s = 0.01599203702 \)

Amount for Millie = \( 1000(1 + 10s) = 1000(1 + (10)(0.1599203702)) = 2599.20 \)
11. You are given that

\[ v(t) = \frac{1}{1 + 0.02t + 0.003t^2} . \]

Calculate \( \delta_{10} \).

**Solution:**

\[ \delta_{10} = \frac{a'(10)}{a(10)} \]

\[ a(t) = \frac{1}{v(t)} = 1 + 0.02t + 0.003t^2 \]

\[ a'(t) = 0.02 + 0.006 t \]

\[ \delta_{10} = \frac{a'(10)}{a(10)} = \frac{0.02 + 0.006(10)}{1 + 0.02(10) + 0.003(10)^2} = \frac{0.08}{1.5} = \frac{0.053333}{1} \]
12. Andrew has the choice of three loans. The loans are identical except for the interest rates. The interest rates on the three loans are:

   a. \( j^{(12)} = 12\% \)
   
   b. A rate equivalent to a nominal annual discount rate of 11.5\% compounded semi-annually.

   c. A force of interest of 11.95\%

Determine which of the above options that Andrew should select and explain why.

**Solution:**

**A.**

\[
(1 + i) = (1 + \frac{j^{(12)}}{12})^{12} = (1 + \frac{.12}{12})^{12} = 1.12682503
\]

**B.**

\[
(1 + i) = (1 - \frac{d^{(2)}}{2})^{-2} = (1 - \frac{.115}{2})^{-2} = 1.125737886
\]

**C.**

\[
(1 + i) = e^{1195} = 1.126933244
\]

Andrew should choose option B because the interest on paying off the loan is lowest. It will take less money to pay off the loan.
13. Trey has invested $10,000 in an account earning a nominal rate of interest of 6.7% compounded monthly.

How much will Trey have at the end of 13 years?

Solution:

\[ AV = 10,000 \left( 1 + \frac{0.067}{12} \right)^{12 \times 13} = 23,835.18 \]