You are given:

a. Mortality follows the Illustrative Life Table.

b. \( i = 6\% \)

Nancy, who is 75 years old, has approached your life insurance company and wants to buy a life annuity due which will pay a benefit of $1,000,000 at the beginning of each month for as long as she lives. Your boss calculates the expected present value of the annuity as \( 1,000,000(\ddot{a}_{75} - 0.5) \) because he assumes that on the average Nancy will die in the middle of the year and subtracting 0.5 adjusts for the death in the middle of the year.

(4 points) Calculate the expected present value that your boss calculated.

\[
EPV = 1,000,000 \left( 7.2170 - 0.50 \right) = 6,717,000
\]

(12 points) Calculate expected present values using three other formulas which will provide a better estimate of the expected present value.

Assuming UDD

\[
\ddot{a}_{75} = \alpha^{(12)} \ddot{a}_{75} - \beta^{(12)} = 1,000,285 \left( 7.2170 \right) - 0.46812 = 6,750,907.6
\]

\[
1,000,000 \left[ 6.75090076 \right] = 6,750,901
\]

Using Common Industry Approximation

\[
\ddot{a}^{0(12)}_{75} = \ddot{a}_{75} - \frac{m-1}{i} = 7.2170 - \frac{11}{2} = 6.7586
\]

\[
1,000,000 \ddot{a}^{0(12)}_{75} = 6,758,667
\]
Using Woolhouse

$$\sigma_{xx} = \frac{m-1}{2m} - \frac{m^2-1}{12m^2} (\mu_x + \delta)$$

$$\mu_{75} = -\frac{1}{2} \left( \ln p_{74} + \ln p_{75} \right)$$

$$= -\frac{1}{2} \left( \ln (1.051269) + \ln (0.94831) \right)$$

$$= 7.2170 - \frac{11}{24} - \frac{143}{1728} \left( \ln (1.06) + 0.050769771 \right) = 0.050769771$$

$$= 6,749,643.211$$

$$1,000,000 \sigma_{75}^{(10)} = 6,749,643$$

(4 points) Recommend an expected value to your boss and explain why it is a better estimate than his estimate.

The Woolhouse formula produces the most accurate estimate so I would recommend

6,749,643.