1. A survivor whole life insurance policy is issued to (70) and (80) who are independent lives. The death benefit is 1,000,000 payable at the end of the year of the second death. The net premiums are paid annually.

You are given:
   a. Mortality follows the Illustrative Life Table.
   b. $i = 6\%$

Calculate the net annual premium for this coverage.

Solution:

$$PVP = PVB$$

$$\overline{P}\ddot{a}_{70:80} = 1,000,000\overline{A}_{70:80}$$

$$P = \frac{1,000,000\overline{A}_{70:80}}{\overline{a}_{70:80}} = \text{using the box formula}$$

$$P = \frac{1,000,000[\overline{A}_{70} + \overline{A}_{80} - \overline{A}_{70:80}]}{\overline{a}_{70} + \overline{a}_{80} - \overline{a}_{70:80}} = \frac{1,000,000[0.51495 + 0.66575 - 0.71690]}{8.5693 + 5.9050 - 5.0014} = 48960.72$$
2. Let the random variable $X$ be the amount of death benefit that a company expects to pay in the next year on a life insurance policy of 100,000.

Assumes that the probability of death in the next year is distributed as follows:

<table>
<thead>
<tr>
<th>Probability of Death</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>0.30</td>
</tr>
<tr>
<td>0.05</td>
<td>0.50</td>
</tr>
<tr>
<td>0.06</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Calculate:

a. $E[X]$

This Material Was Not Covered.

b. $Var[X]$

This Material Was Not Covered
3. A whole life of 100,000 on (75) has a death benefit paid at the end of the year and premium paid annually.

You are given:

a. The premium determined using the equivalence principle is 35,358.90.

b. Mortality follows Mortality Table A.

c. The following Term Structure of Interest Rates:

<table>
<thead>
<tr>
<th>Time</th>
<th>Spot Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.00%</td>
</tr>
<tr>
<td>2</td>
<td>5.00%</td>
</tr>
<tr>
<td>3</td>
<td>6.00%</td>
</tr>
<tr>
<td>4</td>
<td>6.50%</td>
</tr>
<tr>
<td>5</td>
<td>6.75%</td>
</tr>
</tbody>
</table>

Calculate the reserve (policy value) at the end of the second year.

Solution:

We use the recursive method with the forward interest rates.

\[ f_{[0,1]} = y_1 = 4\% \]

\[ 1 + f_{[1,2]} = \frac{(1 + y_2)^2}{1 + y_1} = \frac{(1.05)^2}{1.04} = 1.060096 \implies f_{[1,2]} = 0.060096 \]

\[ _0 V = 0 \]

\[ _1 V = \frac{(_1 V + \delta)(1 + f_{[1,2]}) - S \cdot q_{x+1}}{1 - q_{x+1}} \]

\[ _1 V = \frac{(0 + 35,358.90)(1 + 0.04) - 100,000 \cdot 0.2}{1 - 0.2} = 20,966.57 \]

\[ _2 V = \frac{(20,966.57 + 35,358.90)(1 + 0.060096) - 100,000 \cdot 0.4}{1 - 0.4} = 32,850.68 \]
4. For a multiple decrement table with two decrements, you are given $q_{x}^{(1)} = q_{x}^{(2)} = q_{x}^{(3)} = 0.20$.

Calculate $q_{x}^{(2)}$.

Solution:

\[1 - q_{x}^{(1)} - q_{x}^{(2)} = p_{x}^{(r)}\]

\[p_{x}^{(1)} \cdot p_{x}^{(2)} = p_{x}^{(r)}\]

\[\therefore 1 - q_{x}^{(1)} - q_{x}^{(2)} = p_{x}^{(1)} \cdot p_{x}^{(2)} = (1 - q_{x}^{(1)})(1 - q_{x}^{(2)})\]

\[1 - 0.2 - 0.2 = (1 - 0.2)(1 - q_{x}^{(2)})\]

\[0.6 = 1 - q_{x}^{(2)} \implies q_{x}^{(2)} = 0.25\]
5. A whole life insurance policy on (65) for 250,000 pays a death benefit immediately upon death. The premiums are paid annually.

You are given:
   a. Mortality follows the Illustrative Life Table.
   b. \( i = 6\% \)
   c. Deaths are uniformly distributed between integer ages.

Calculate the net premium reserve at the end of 10.25 years.

Solution:

\[
10.25\ V = (0.75)_{10}V + P + (0.25)_{11}V
\]

\[
P = \frac{250,000\bar{A}_{65}}{\bar{a}_{65}} = \frac{(250,000)(0.43980)(1.02971)}{9.8969} = 11,439.60
\]

\[
_{10}V = PVFB - PVFP = 250,000\bar{A}_{65} - (11,439.60)\bar{a}_{65} =
\]

\[
(250,000)(0.59149)(1.02971) - (11,439.60)(7.217) = 69,706.17
\]

\[
_{11}V = PVFB - PVFP = 250,000\bar{A}_{66} - (11,439.60)\bar{a}_{66} =
\]

\[
(250,000)(0.60665)(1.02971) - (11,439.60)(6.9493) = 76,671.15
\]

\[
_{10.25}V = (0.75)(69,706.17 + 11,439.60) + (0.25)(76,671.15) = 80,027.12
\]
6. For a multiple state model, there are three states:

   i. State 0 is a person is healthy
   ii. State 1 is a person is sick
   iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Also, a person in State 1 can transition to State 0 or to State 2. Finally, a person in State 2 cannot transition.

You are given the following transitional intensities:

   i. \( \mu_{01} = 0.12(t + 1) \)
   ii. \( \mu_{10} = 0.012 \)
   iii. \( \mu_{02} = 0.024 \)
   iv. \( \mu_{12} = 0.048t \)

Assume that only one transition can occur in any one month period.

An insurance company starts with 1000 healthy lives at time 0. Calculate the number of healthy lives at the end of 2 months.

**Solution:**

As I look at this problem a year later, I think it probably is somewhat confusing. t should be in years which I am not sure is clear.
Create a Transition Matrix

\[
Q_t = \begin{bmatrix}
1 - \left(\frac{1}{12}\right)0.12(t+1) - \left(\frac{1}{12}\right)0.024 & \left(\frac{1}{12}\right)0.12(t+1) & \left(\frac{1}{12}\right)0.024 \\
\left(\frac{1}{12}\right)0.012 & 1 - \left(\frac{1}{12}\right)0.012 - \left(\frac{1}{12}\right)0.048t & \left(\frac{1}{12}\right)0.048t \\
0 & 0 & 1
\end{bmatrix}
\]

\[
Q_0 = \begin{bmatrix}
1 - \left(\frac{1}{12}\right)0.12(1) - \left(\frac{1}{12}\right)0.024 & \left(\frac{1}{12}\right)0.12(1) & \left(\frac{1}{12}\right)0.024 \\
\left(\frac{1}{12}\right)0.012 & 1 - \left(\frac{1}{12}\right)0.012 - \left(\frac{1}{12}\right)0.048(0) & \left(\frac{1}{12}\right)0.048(0) \\
0 & 0 & 1
\end{bmatrix}
\]

\[
Q_{1/12} = \begin{bmatrix}
1 - \left(\frac{1}{12}\right)0.12(1/12 + 1) - \left(\frac{1}{12}\right)0.024 & \left(\frac{1}{12}\right)0.12(1/12 + 1) & \left(\frac{1}{12}\right)0.024 \\
\left(\frac{1}{12}\right)0.012 & 1 - \left(\frac{1}{12}\right)0.012 - \left(\frac{1}{12}\right)0.048(1/12) & \left(\frac{1}{12}\right)0.048(1/12) \\
0 & 0 & 1
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0.9871667 & 0.0108333 & 0.002 \\
0.001 & 0.9986667 & 0.0003333 \\
0 & 0 & 1
\end{bmatrix}
\]

We can do this using matrix multiplication, or the tree method, or using probability table. I will use probability table.

**Time 0   Time 1   Time 2**

**Status 0   Status 0   Status 0 ==> (0.988)(0.98716667) = 0.97532**

**Status 0   Status 1   Status 0 ==> (0.01)(0.001) = 0.00001**

\[p_1^\infty = 0.97532 + 0.00001 = 0.97533\]

*Heathly Lives = (1000)(0.97533) = 975.33*
7. You are given:
   a. There are two decrements in a multiple decrement model.
   b. Each decrement is uniformly distributed in the associated single decrement table.
   c. \( q^{(1)}_s = 0.08 \)
   d. \( q^{(1)}_s' = 0.10 \)

Calculate \( q^{(2)}_s \).

Solution:

*Under UDD in the Single Decrement Table*

\[
q^{(1)}_s = q^{(1)}_s[1 - 0.5q^{(2)}_s] \quad \text{and} \quad q^{(2)}_s = q^{(2)}_s[1 - 0.5q^{(1)}_s]
\]

\[
q^{(1)}_s = q^{(1)}_s[1 - 0.5q^{(2)}_s] \Rightarrow 0.08 = 0.10[1 - 0.5q^{(2)}_s] \Rightarrow q^{(2)}_s = 0.4
\]

\[
q^{(2)}_s = 0.4[1 - 0.5(0.10)] = 0.38
\]
8. A whole life policy of 5000 on (75) has a death benefit payable at the end of the year of death and an annual premium.

You are given the following:
   a. Mortality follows Mortality Table A.
   b. $i = 10\%$
   c. The annual premium is 3000.
   d. The following expenses are incurred:

<table>
<thead>
<tr>
<th>Expense Type</th>
<th>Beginning of First Year</th>
<th>Beginning of each Year 2 +</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent of Premium</td>
<td>60%</td>
<td>8%</td>
</tr>
<tr>
<td>Per Policy</td>
<td>100</td>
<td>40</td>
</tr>
</tbody>
</table>

e. An expense of 50 is incurred at the end of the year of death.

Calculate the asset share at the end of the second year.

Solution:

\[ AS_0 = 0 \]

\[ AS_{t+1} = \frac{(AS_t + P_t - e_t - X_t^{ROY})(1 + i) - (q_{x+t})(S_{t+1} + E_{t+1})}{p_{x+t}} \]

\[ AS_1 = \frac{(0 + 3000(1 - 0.6) - 100)(1.1) - (0.2)(5000 + 50)}{0.8} = 250 \]

\[ AS_2 = \frac{(250 + 3000(1 - 0.08) - 40)(1.1) - (0.4)(5000 + 50)}{0.6} = 2078.33 \]
9. In a multiple decrement table with two decrements, you are given:
   a. \( q^{(1)}_x = 0.24 \)
   b. \( q^{(2)}_x = 0.10 \)
   c. Decrement (1) occurs uniformly between integer ages in the associated single decrement table.
   d. Decrement (2) occurs at age \( x + 0.75 \)

Calculate \( q^{(1)}_x \)

Solution:

\[
q^{(1)}_x = \frac{d^{(1)}_x}{L^{(x)}_x}; \quad \text{Let } L^{(x)}_x = 1000.
\]

If there were no other decrements, then the number of decrements would be:

\[1000q^{(1)}_x = 1000(0.24) = 240 \quad \text{and these would occur uniformly through the year}\]

which means that 180 would occur from 0 to 0.75 and 60 would occur from 0.75 to 1.

Decrement 1 occurs at time 0.75 so there are no other decrements during time 0 to 0.75

so \( 0.75d_x = 180 \). Then at time 0.75, 10% of the people leave because of decrement (2).

Therefore, there are only 90% of the population left for the last quarter of the year to be exposed to decrement 1 so then \( 0.25d_{x+0.75} = (0.90)(60) = 54 \).

\[\Rightarrow q^{(1)}_x = \frac{d^{(1)}_x}{L^{(x)}_x} = \frac{180 + 54}{1000} = 0.234\]
10. You are given:
   a. Mortality follows Mortality Table A.
   b. The following Term Structure of Interest Rates:

<table>
<thead>
<tr>
<th>Time</th>
<th>Spot Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.00%</td>
</tr>
<tr>
<td>2</td>
<td>5.00%</td>
</tr>
<tr>
<td>3</td>
<td>6.00%</td>
</tr>
<tr>
<td>4</td>
<td>6.50%</td>
</tr>
<tr>
<td>5</td>
<td>6.75%</td>
</tr>
</tbody>
</table>

Calculate the net annual premium for joint whole life which pays a death benefit of 50,000 upon the first death of (77) and (78). (77) and (78) are independent lives. The death benefit is paid at the end of the year and premium is paid annually.

Solution:

Do it with \( l_x \)'s

<table>
<thead>
<tr>
<th></th>
<th>( l_x ) for 77</th>
<th>( l_x ) for 78</th>
<th>( l_x ) for 77:78</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age 77:78</td>
<td>960</td>
<td>480</td>
<td>460,800</td>
</tr>
<tr>
<td>Age 78:79</td>
<td>480</td>
<td>120</td>
<td>57,600</td>
</tr>
<tr>
<td>Age 79:80</td>
<td>120</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
PVP = PVB \\
P(460,800 + 57,600(1.04)^{-1}) = (50,000)\{[468,800 - 57,600](1.04)^{-1} + [57,600 - 0](1.05)^{-2}\} \\
P = \frac{(50,000)\{[468,800 - 57,600](1.04)^{-1} + [57,600 - 0](1.05)^{-2}\}}{460,800 + 57,600(1.04)^{-1}} = 42,614.33
\]
11. A whole life policy of 25,000 on (40) has a death benefit payable at the end of the year of death and an annual premium.

You are given the following:

a. Mortality follows the Illustrative Life Table
b. $i = 6\%$
c. The gross annual premium is calculated as 110% of the gross premium that would be determined using the equivalence principle.
d. The following expenses are incurred:

<table>
<thead>
<tr>
<th>Expense Type</th>
<th>Beginning of First Year</th>
<th>Beginning of each Year 2 +</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent of Premium</td>
<td>60%</td>
<td>8%</td>
</tr>
<tr>
<td>Per Policy</td>
<td>400</td>
<td>40</td>
</tr>
</tbody>
</table>

e. An expense of 100 is incurred at the end of the year of death.

Calculate the gross premium reserve at the end of the 20th year.

Solution:

*Find Premium using the equivalence principle* \(\Rightarrow PVP=PVB+PVE\)

\[
P \cdot \ddot{a}_{40} = 25,000A_{40} + 0.52P + 0.08P \cdot \ddot{a}_{40} + 360 + 40\ddot{a}_{40} + 100A_{40}
\]

\[
P = \frac{25,100A_{40} + 360 + 40\ddot{a}_{40}}{0.92\ddot{a}_{40} - 0.52} = \frac{(25,100)(0.16132) + 360 + (40)(14.8166)}{(0.92)(14.8166) - 0.52} = 381.48823
\]

\[
G = (1.1)(P) = (1.1)(381.48823) = 419.64
\]

\[
_20V = PVFB + PVFE - PVFP =
\]

\[
25,000A_{40} + (0.08)(419.64)\ddot{a}_{40} + 40\ddot{a}_{40} + 100A_{40} - (419.64)\ddot{a}_{40} =
\]

\[
(25,100)(0.36913) + (40)(11.1454) - (0.92)(419.64)(11.1454) = 5408.09
\]
12. For a multiple state model, there are three states:

i. State 0 is a person is healthy
ii. State 1 is a person is permanently disabled
iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Further, a person in State 1 can transition to State 2, but not to State 0. Finally, a person in State 2 cannot transition.

You are given the following transitional intensities:

i. \( \mu_{01} = 0.08 \)
ii. \( \mu_{02} = 0.03 \)
iii. \( \mu_{12} = 0.10 \)

Calculate \( p^{02}_x \).

Solution:

\[
\begin{align*}
\quad p^{02}_x &= 1 - p^{00}_x - p^{01}_x \\
\quad p^{00}_x &= e^{-\int_{0}^{4} (0.08+0.03) \, ds} = e^{-0.44} = 0.64404 \\
\quad p^{01}_x &= e^{-\int_{0}^{4} (0.08+0.03) \, ds} = e^{-0.44} = 0.64404 \\
\quad p^{12}_x &= \int_{0}^{4} e^{-0.11s} \, ds = 0.08 \int_{0}^{4} e^{-0.11s} \, ds = 0.08 \int_{0}^{4} e^{-0.11s} \cdot e^{-0.10(4-s)} \, ds = \\
&= 0.08 \int_{0}^{4} e^{-0.40-0.01s} \, ds = (0.08)(e^{-40})(1-e^{-0.04}) = 0.210269 \\
\quad p^{02}_x &= 1 - p^{00}_x - p^{01}_x = 1 - 0.64404 - 0.210269 = 0.14569
\end{align*}
\]
13. A 20 year increasing term plan on (70) has a death benefit that increases every year. The death benefit is year \( k \) is \( 1000k \) for \( k = 1, 2, \ldots, 20 \). In other words, the death benefit in the first year is 1000. The death benefit in the second year is 2000. The death benefit continues to increase until the death benefit is 20,000 in the 20th year. The death benefit is payable at the end of the year of death.

You are given:
   a. Mortality follows the Illustrative Life Table.
   b. \( i = 6\% \)
   c. The net annual premium is 505.17.
   d. The net premium reserve at the end of the 10 year is 5348.57.

Calculate the net premium reserve at the end of the 11th year.

Solution:

\[
11V = \frac{(10V + P)(1 + i) - S_{11} \cdot q_{70+10}}{p_{70+10}} = \frac{(5348.57 + 505.17)(1.06) - (1000)(11)(0.0803)}{(1 - 0.0803)} = 5786.30
\]
14. A whole life insurance policy on (80) for 10,000 pays a death benefit at the end of the year of death. The premiums are paid annually.

You are given:

a. Mortality follows the Illustrative Life Table.

b. \( i = 6\% \)

c. \( P_{FPT} \) is the first year premium under the Full Preliminary Term reserve method.

d. \( P_{x+1} \) is the renewal premium for years 2 and later under Full Preliminary Term.

e. The expense allowance is defined as \( P_{x+1} - P_{FPT} \).

Calculate the expense allowance for this policy.

Solution:

\[ P_{FPT} = 10,000vq_{80} = (10,000)(1.06)^{-1}(0.0803) = 757.54717 \]

\[ P_{x+1} = \frac{10,000A_{81}}{\bar{a}_{81}} = \frac{(10,000)(0.680)}{5.6533} = 1202.83728 \]

\[ Expense\ Allowance = 1202.83728 - 757.54717 = 445.29 \]

Calculate the reserve under Full Preliminary Term at the end of the 5\(^{th}\) year.

Solution:

\[ sV = PVFB - PVFP = 10,000A_{85} - P_{x+1} \cdot \bar{a}_{85} = (10,000)(0.73407) - (1202.83728)(4.698) = 1689.77 \]
15. In a multiple decrement table, you are given:
   a. Decrement 1 is death.
   b. Decrement 2 is diagnosis of critical illness.
   c. Decrement 3 is lapse.

Further, you are given the following multiple decrement table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$q_x^{(1)}$</th>
<th>$q_x^{(2)}$</th>
<th>$q_x^{(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>77</td>
<td>0.25</td>
<td>0.10</td>
<td>0.15</td>
</tr>
<tr>
<td>78</td>
<td>0.50</td>
<td>0.05</td>
<td>0.20</td>
</tr>
<tr>
<td>79</td>
<td>0.90</td>
<td>0.10</td>
<td>0.00</td>
</tr>
</tbody>
</table>

An whole life insurance policy on (77) pays a benefit of 100,000 upon death. It also pays a benefit of 50,000 upon diagnosis of critical illness. No benefit is paid upon lapse. All benefits are paid at the end of the year of death.

You are given that $\nu = 0.90$.

The premium is paid annually.

Calculate the annual premium for this policy.

Solution:

Doing it with $l_x^{(r)}$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$l_x^{(3)}$</th>
<th>$d_x^{(1)}$</th>
<th>$d_x^{(2)}$</th>
<th>$d_x^{(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>77</td>
<td>10,000</td>
<td>2500</td>
<td>1000</td>
<td>1500</td>
</tr>
<tr>
<td>78</td>
<td>5000</td>
<td>2500</td>
<td>250</td>
<td>1000</td>
</tr>
<tr>
<td>79</td>
<td>1250</td>
<td>1125</td>
<td>125</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
PVP = PVB
\]

\[
P[10,000 + 5000(0.9) + 1250(0.9)^2] = (100,000)[2500(0.9) + 2500(0.9)^2 + 1125(0.9)^3] + (50,000)[1000(0.9) + 250(0.9)^2 + 125(0.9)^3]
\]

\[
P = 36,692.59
\]
16. A joint annuity pays 10,000 at the beginning of each year if both (40) and (50) are alive. The annuity only pays 4000 at the beginning of each year if (40) is alive and (50) is dead. Additionally, the annuity pays 5000 at the beginning of each year if (50) is alive and (40) is dead.

The two lives are independent.

You are given that mortality follows the Illustrative Life Table and \( i = 6\% \).

Calculate the expected present value for this annuity.

Solution:

\[ PV = a \cdot \dd{a}_{40} + b \cdot \dd{a}_{50} + c \cdot \dd{a}_{40:50} \]

\( a = \text{amount payable when 40 is alive} = 4000 \)

\( b = \text{amount payable when 50 is alive} = 5000 \)

\( a + b + c = \text{amount payable when both are alive} = 10,000 \Rightarrow c = 10,000 - 5000 - 4000 = 1000 \)

\[ PV = (4000)\dd{a}_{40} + (5000)\dd{a}_{50} + (1000)\dd{a}_{40:50} = (4000)(14.8166) + (5000)(13.2668) + (1000)(12.4784) = 138,078.80 \]