1. (8 points) You are given:
   a. \( 1000A_y = 500 \)
   b. \( a_{s+1} = 3.62963 \)
   c. \( d = 0.10 \)

   Calculate \( p_s \).

   **Solution:**

   \( 1000A_y = 500 \implies A_y = 0.500 \)

   \[
   \ddot{a}_i = \frac{1 - A_y}{d} = \frac{1 - 0.500}{0.1} = 5
   \]

   \[
   \ddot{a}_{s+1} = a_{s+1} + 1 = 3.62963 + 1 = 4.62963
   \]

   \[
   \ddot{a}_s = 1 + v \cdot p_s \cdot \ddot{a}_{s+1} \implies 5 = 1 + (1 - 0.1)p_s(4.62963)
   \]

   \[
   \therefore p_s = \frac{5 - 1}{(0.9)(4.62963)} = 0.96
   \]
2. (8 points) Muhammad will receive a 10 year certain and life annuity when he becomes 65. The annuity will pay an annual benefit of 100,000 with the first payment at age 65. The first 10 payments are guaranteed. Payments thereafter are only paid if Muhammad is alive.

You are given:
   a. Mortality follows the Illustrative Life Table
   b. \(i = 6\%\)

Calculate the Expected Present Value at age 65 of Muhammad’s annuity.

Solution:

\[
EPV = 100,000\bar{a}_{65:10} = 100,000(\bar{a}_{65:10} + 10 \bar{d}_{65}) = \\
100,000\left(\frac{1 - v^{10}}{i} + 10 E_{65} \cdot \bar{a}_{75}\right) = 100,000\left(\frac{1 - (1.06)^{-10}}{0.06 / 1.06} + (0.39994)(7.2170)\right) = \\
1,068,806
\]
3. Pestka Pet Insurance Company has developed the following life insurance table for dogs:

<table>
<thead>
<tr>
<th>Age</th>
<th>( l_x )</th>
<th>Age</th>
<th>( l_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2000</td>
<td>5</td>
<td>1200</td>
</tr>
<tr>
<td>1</td>
<td>1950</td>
<td>6</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>1850</td>
<td>7</td>
<td>700</td>
</tr>
<tr>
<td>3</td>
<td>1600</td>
<td>8</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>1400</td>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>

Pestka sells a pet insurance policy on a dog who is age 5. The policy pays a death benefit at the end of the year of death of 1000. Level annual premiums are payable for the life of the dog.

The expenses associated with the policy are 10% of premium at the beginning of each year.

The interest rate used to calculate all values is 4%.

A. (7 points) Calculate the gross premium for this insurance using the Equivalence Principle.

**Solution:**

\[
PVP = PVB + PVE
\]

\[
P(1200 + 1000v + 700v^2 + 300v^3) = 1000(200v + 300v^2 + 400v^3 + 300v^4) + 0.1P(1200 + 1000v + 700v^2 + 300v^3)
\]

\[
P = \frac{1000(200v + 300v^2 + 400v^3 + 300v^4)}{0.9(1200 + 1000v + 700v^2 + 300v^3)} = 390.81
\]

B. (5 points) Calculate the loss that Pestka will incur if the dog dies in the second year if the gross premium is calculated using the Equivalence Principle.

\[
L_0^x = 1000v^{K_x+1} + 0.1(390.81)\dd{a}_{K_x+1} - (390.81)\dd{a}_{K_x+1} = 1000v^{K_x+1} - 0.9(390.81)\dd{a}_{K_x+1}
\]

For \( K_x = 1 \) which means the dog dies in the second year

\[
L_0^x = 1000v^{1+1} - 0.9(390.81)\dd{a}_{1+1} = 234.63
\]
C. (5 points) Petska decides to charge a gross premium so that the loss will be zero if the dog dies in the second year. Determine the gross premium that Petska decides to charge.

We want \( L_0^+ = 0 \) for \( k = 1 \)

\[
L_0^+ = 1000v^{1+1} - 0.9(P)\widetilde{a}_{\overline{2}|1} = 0
\]

\[
P = \frac{1000v^2}{0.9(\widetilde{a}_{\overline{2}|1})} = 523.71
\]

D. (7 points) Estimate the monthly premium equivalent to the annual premium in Part C. by using the Woolhouse formula.

\[
(P^{\text{Annual}})\widetilde{a}_5 = (P^{\text{Monthly}})(12)(\widetilde{a}_5^{(12)})
\]

\[
(523.71)(\widetilde{a}_5) = (P^{\text{Monthly}})(12)(\widetilde{a}_5^{(12)}) = \frac{12 - 1}{(2)(12)} - \frac{12^2 - 1}{(12)(12^2)} \left[ \frac{0.04}{1.04} - 0.5 \left[ \ln \left( \frac{1200}{1400} \right) + \ln \left( \frac{1000}{1200} \right) \right] \right]
\]

\[
\widetilde{a}_5 = \frac{(1200 + 1000v + 700v^2 + 300v^3)}{1200} = 2.56285599
\]

\[
P^{\text{Monthly}} = \frac{(523.71)(2.56285599)}{(12)(2.56285599) - \frac{12 - 1}{(2)(12)} - \frac{12^2 - 1}{(12)(12^2)} \left[ \frac{0.04}{1.04} - 0.5 \left[ \ln \left( \frac{1200}{1400} \right) + \ln \left( \frac{1000}{1200} \right) \right] \right]}
\]

\[
\frac{1342.19}{25.04900493} = 53.58
\]
4. (8 points) $Y$ is the present value random variable for a temporary 10 year life annuity due to (80) which pays 1000 annually.

You are given:

a. Mortality follows the Illustrative Life Table
b. $i = 6\%$

Calculate the $Var(Y)$

Solution:

$$Var(Y) = (1000)^2 \left( \frac{2A_{0.10} - (A_{0.10})^2}{d^2} \right) = (1000)^2 \left( \frac{2A_{0.10} - v^{10}_{10} E_{80} - \theta^2 A_{90} + v^{10}_{10} E_{90}) - (A_{80} - \theta E_{80} \cdot A_{90} + \theta E_{90})^2}{d^2} \right)$$

$$= (1000)^2 \left( 0.47359 \cdot (1.06)^{10} (0.15100)(0.64496) + (1.06)^{10} (0.15100) \right) - \left( \frac{0.66575 \cdot (0.15100)(0.79346) + 0.15100}{(0.05660)^2} \right)$$

$$= 5,557,627$$
5. You are given:
   a. Mortality follows the Illustrative Life Table
   b. \( i = 6\% \)

Blink Life Insurance Company sells whole life insurance policies of 250,000 to lives age 25. The death benefit is payable at the end of the year of death and level annual premiums are payable for the life of the insured.

Blink pays commissions of 100% of premiums in the first year and 10% of premiums thereafter. Blink also has a 75 per policy expense at the start of each year.

Blink hired two consulting actuaries to calculate the level annual premium that should be charged for this policy.

A. (8 points) Ben Jin from Chee Consultants Inc. calculated the premium using the Equivalence Principle. Determine the premium that Ben Jin calculated.

\[
PVP = PVB + PVE \\

P\ddot{a}_{25} = 250,000A_{25} + 0.9P + 0.1P\ddot{a}_{25} + 75\ddot{a}_{25} \\
P = \frac{250,000A_{25} + 75\ddot{a}_{25}}{\ddot{a}_{25} - 0.1\ddot{a}_{25} - 0.9} = \frac{250,000(0.08165) + 75(16.2242)}{16.2242(0.9) - 0.9} = 1578.58
\]

B. (16 points) Peter from Peter’s Perfect Premiums determined the premium using the Portfolio Percentile Premium Principle assuming that Blink would sell 10,000 policies. The premium was determined so that there was a 95% chance of achieving a profit. Determine the premium that Peter calculated.
6. (8 points) Hassan, age 35, purchases a whole life policy with a death benefit of 1 million payable at the moment of death. He will pay level annual premiums during his lifetime.

You are given:

a. Mortality follows the Illustrative Life Table
b. \( i = 6\% \)
c. Death are uniformly distributed between integer ages.
d. Expenses occurring at the beginning of the year are:
   i. First year expenses of 800 per policy and 2 per 1000 of death benefit.
   ii. Renewal expenses (occurring at the start of the second year and every year thereafter) are 60 per policy.
   iii. Commissions of 70% of premiums in the first year and 8% of premiums thereafter.
   iv. Premium Tax of 3% of premium.
e. The company will incur a termination expense at the moment of death of 1000.

Calculate the gross premium for this policy using the Equivalence Principle.

Solution:

\[
PVP = PVB + PVE
\]

\[
P \ddot{u}_{35} = 1,000,000 \ddot{A}_{35} + 740 + (1000)2 + 60 \ddot{a}_{35} + 0.62P + 0.08P \ddot{a}_{35} + 0.03P \ddot{a}_{35} + 1000 \ddot{A}_{35}
\]

\[
P = \frac{1,000,000 \ddot{A}_{35} + 740 + (1000)2 + 60 \ddot{a}_{35} + 1000 \ddot{A}_{35}}{\ddot{a}_{35} - 0.62 - 0.08 \ddot{a}_{35} - 0.03 \ddot{a}_{35}} =
\]

\[
\frac{1,001,000 \ddot{A}_{35} + 2,740 + 60 \ddot{a}_{35}}{(0.89)\ddot{a}_{35} - 0.62} = \frac{1,001,000(1.02971)(0.12872) + 2,740 + 60(15.3926)}{(0.89)(15.3926) - 0.62} =
\]

\[
10,424.04
\]
7. Rafidah purchases a whole life insurance policy with a death benefit of 50,000 payable at the moment of death. Level premiums are paid continuously for as long as Rafidah is alive.

You are given:
   a. Rafidah is now age 50.
   b. Mortality follows the Illustrative Life Table
   c. \( i = 6\% \)
   d. Deaths are uniformly distributed between integer ages.

A. (8 points) Calculate the annual net premium rate paid by Rafidah using the equivalence principle.

Solution:

\[
PVP = PVB
\]

\[
\bar{P}_{50} = 50,000\overline{A}_{50} \implies P = \frac{50,000\overline{A}_{50}}{\overline{a}_{50}} = \frac{50,000\overline{A}_{50}}{1 - \overline{A}_{50}} \cdot \frac{\delta}{1 - (i / \delta)\overline{A}_{50} \cdot \delta} =
\]

\[
\frac{50,000(0.06)(0.24905)}{1 - (1.02971)(0.24905)} = 1004.84
\]

B. (8 points) Calculate the Variance for the loss random variable, \( L^\alpha \), for this policy.

Solution:

\[
Var = \left( S + \frac{P}{\delta} \right)^2 \left( \overline{\overline{A}}_{50} - \overline{A}_{50} \right)^2 = \left( 50,000 + \frac{1004.84}{0.05827} \right)^2 \left( \frac{(1.06)^2 - 1}{2(0.05827)} \right) 0.09476 - (1.02971)^2(0.24905)^2 =
\]

157,062,748.5
8. (8 points) You are given:
   
a. \( t_P = e^{-0.05t} \)

b. \( \delta = 0.03 \)

Calculate \( \bar{a}_x \).

Solution:

\[
\bar{a}_x = \int_0^\infty e^{-\delta t} \cdot t_P \cdot dt = \int_0^\infty e^{-0.03t} \cdot e^{-0.05t} \cdot dt = \int_0^\infty e^{-0.08t} \cdot dt = \frac{1 - 0}{0.08} = 12.5
\]