1. Kunyu who is (20) purchases a special endowment insurance policy.

The policy pays a death benefit of 10,000 if Kunyu dies before age 40. It pays a death benefit of 25,000 if Kunyu dies between age 40 and 60. The death benefit is payable at the moment of death.

If Kunyu lives to age 60, an endowment benefit of 25,000 will be paid.

You are given:
   a. Mortality follows the Illustrative Life Table.
   b. \( i = 6\% \)
   c. Deaths are uniformly distributed between integral ages.

Calculate the Actuarial Present Value of Kunyu's policy.

**Solution**

\[
APV = 10,000 \bar{A}_{20:40} + 15,000 \bar{A}_{20:40} + 25,000_{40} E_{20} = 10,000 \left( \frac{i}{\delta} \right) A_{20:40} + 15,000 \left( \frac{i}{\delta} \right) E_{20} A_{40:20} + 25,000_{40} E_{20} = 
\]

\[
10,000 \left( \frac{i}{\delta} \right) (A_{20:40} E_{20} - A_{40}) + 15,000 \left( \frac{i}{\delta} \right) (E_{20} A_{20:40} - A_{40} E_{20} A_{40}) + 25,000_{40} E_{20} = 
\]

\[
10,000 (1.02971)(0.06528 - (0.30193)(0.27414)(0.36913)) + 15,000 (1.02971)(0.30193)(0.16132 - (0.27414)(0.36913)) + 25,000(0.30193)(0.27414) = 2707.26
\]

OR

\[
APV = 25,000 \bar{A}_{20:40} - 15,000 \bar{A}_{20:30} = 25000(\bar{A}_{20:40} + E_{20}) - 15,000 \bar{A}_{20:30} = 
\]

\[
25000 \left( \frac{i}{\delta} \right) (A_{20:40} E_{20} + E_{20}) - 15,000 \left( \frac{i}{\delta} \right) (A_{20:20} E_{20} A_{40}) = 
\]

\[
25000(1.02971)(0.06528 - (0.30193)(0.27414)(0.36913)) + (0.30193)(0.27414)) - 15,000(1.02971)(0.06528 - (0.30193)(0.16132)) = 2707.26
\]
2. Wang Insurance Company issues 900 policies which provide term insurance coverage until age 90. The lives covered by these 900 policies are independent lives. Each policy is sold to (50) and provides a death benefit of 25,000 payable at the end of the year of death.

You are given:
   a. Mortality follows the Illustrative Life Table.
   b. \( i = 6\% \)

Assuming the normal distribution, calculate the amount that Wang must have on hand at time 0 to be 97.5% certain that the company can cover the future death benefits.

**Solution**

\[ E[Z] = 25,000 \cdot A_{50.40}^{1} \]

\[ A_{50.40}^{1} = (A_{50} - v_{40}^{40} \cdot E_{50}) = (0.24905 - (0.23047)(0.04988)(0.79346)) = 0.239929 \]

\[ \Rightarrow E[Z] = 25,000 \cdot A_{50.40}^{1} = 5998.23 \]

\[ Var[Z] = 25,000^2 \left( 2 \cdot A_{50.40}^{1} - (A_{50.40}^{1})^2 \right) \]

\[ 2 \cdot A_{50.40}^{1} = (2 \cdot A_{50} - v_{40}^{40} \cdot E_{50} \cdot 2 \cdot A_{50}) = (0.09476 - (0.097222)(0.23047)(0.04988)(0.64496)) = 0.094039 \]

\[ Var[Z] = 25,000^2 \left( 2 \cdot A_{50.40}^{1} - (A_{50.40}^{1})^2 \right) = 25,000^2 \left( 0.094039 - (0.239929)^2 \right) = 22,795,571.85 \]

\[ E[Port] = (900)(5998.23) = 5,398,407 \]

\[ Var[Port] = (900)(22,795,571.85) \]

\[ Amount\ Needed = E[Port] + 1.96 \sqrt{Var[Port]} = 5,398,407 + 1.96 \sqrt{(900)(22,795,571.85)} = 5,679,147 \]
1. Kunyu who is (20) purchases a special endowment insurance policy.

The policy pays a death benefit of 10,000 if Kunyu dies before age 40. It pays a death benefit of 25,000 if Kunyu dies between age 40 and 60. The death benefit is payable at the moment of death.

If Kunyu lives to age 60, an endowment benefit of 25,000 will be paid.

You are given:
   a. Mortality follows the Illustrative Life Table.
   b. $i = 6\%$
   c. Deaths are uniformly distributed between integral ages.

Calculate the Actuarial Present Value of Kunyu’s policy.

Solution

\[ APV = 10,000 \ddot{A}_{\overline{1}\,_{20:40}} + 15,000 \ddot{A}_{\overline{1}\,_{20:40}} + 25,000 \cdot E_{20} = 10,000 \left( \frac{i}{\delta} \right) \ddot{A}_{\overline{1}\,_{20:40}} + 15,000 \left( \frac{i}{\delta} \right) (E_{20} \ddot{A}_{\overline{1}\,_{40:60}}) + 25,000 \cdot E_{20} = \]

\[ 10,000 \left( \frac{i}{\delta} \right) (A_{20} - 40 \cdot E_{20} \cdot A_{40}) + 15,000 \left( \frac{i}{\delta} \right) (E_{20} \cdot A_{40} - 20 \cdot E_{40} \cdot A_{50}) + 25,000 \cdot E_{20} = \]

\[ 10,000 (1.02971)(0.06528 - (0.30193)(0.27414)(0.36913)) \]
\[ + 15,000 (1.02971)(0.30193)(0.16132 - (0.27414)(0.36913)) + 25,000(0.30193)(0.27414) = 2707.26 \]

OR

\[ APV = 25,000 \ddot{A}_{\overline{1}\,_{20:40}} - 15,000 \ddot{A}_{\overline{1}\,_{20:40}} = 25000(\ddot{A}_{\overline{1}\,_{20:40}} + E_{20}) - 15,000 \ddot{A}_{\overline{1}\,_{20:40}} = \]

\[ 25000 \left[ \frac{i}{\delta} \right] (A_{20} - 40 \cdot E_{20} \cdot A_{40}) + 40 \cdot E_{20} - 15,000 \left[ \frac{i}{\delta} \right] (A_{20} - 20 \cdot E_{20} \cdot A_{50}) = \]

\[ 25000 \left[ 1.02971 \right] (0.06528 - (0.30193)(0.27414)(0.36913)) + (0.30193)(0.27414) \]
\[ - 15,000 \left[ 1.02971 \right] (0.06528 - (0.30193)(0.16132)) = 2707.26 \]
2. \( Z \) is the present value random variable for a whole life insurance issued to (70) where the death benefit is 1000 payable at the moment of death.

You are given:
   a. Mortality follows the Illustrative Life Table.
   b. \( i = 5\% \)
   c. Deaths are uniformly distributed between integral ages.

Calculate the \( \Pr(Z < 500) \).

Solution

\[ Z = 1000v^x \]

\[ \Pr(Z < 500) = \Pr(1000v^x < 500) = \Pr(v^x < 0.5) = \Pr \left( T_x > \frac{\ln(0.5)}{\ln(v)} \right) = \Pr(T_x > 14.2067) = \]

\[ 14.2067 P_{70} = \frac{L_{84,2007}}{L_{70}} \quad \text{Since UDD} \quad \Rightarrow \quad \frac{L_{84}(1 - 0.2067) + L_{85}(0.2067)}{L_{70}} = \]

\[ \frac{(2,660,734)(1 - 0.2067) + (2,358,246)(0.2067)}{6,616,155} = 0.3927 \]