1. A fully discrete whole life policy to (60) has a death benefit of 20,000 and an annual gross premium of 865.

Mortality follows the Illustrative Life table with $i = 0.05$. Note that the interest rate is NOT 6%.

The policy incurs the following expenses:
   i. 30 per policy at the beginning of each policy year.
   ii. 50% of premium during the first policy year.
   iii. 10% of premium during each year after the first policy year.

Calculate the Asset Share at the end of the second year.

\[
\begin{align*}
A_S &= 0 \\
A_{S_1} &= \frac{(0 + 865 - 30 - 0.5 \times 865) \times (1 + 0.05) - 20000 \times 0.01376}{1 - 0.01376} \\
&= 149.48187 \\
A_{S_2} &= \frac{(149.48187 + 865 - 30 - 0.1 \times 865) \times (1 + 0.05) - 20000 \times 0.01501}{1 - 0.01501} \\
&= 652.4746
\end{align*}
\]
2. A fully discrete 20 year endowment insurance policy to (40) has a death benefit of 25,000 and annual premiums for 20 years.

You are given that mortality follows the Illustrative Life Table and \( i = 0.06 \).

Calculate \( \nu_{10}^{-} \), the net benefit reserve at the end of 10 years.

\[
\nu_{40:20} \approx 25000 \times A_{40:20}
\]

\[
P = \frac{25000 \times (A_{40} - 20E_{40}A_{60} + 20E_{40})}{\ddot{a}_{40} - 20E_{40} \ddot{a}_{60}}
\]

\[
= \frac{25000 \times (0.16132 - 0.27414 \times 0.36913 + 0.27414)}{14.8166 - 0.27414 \times 11.1454}
\]

\[
= 710.5285
\]

\[
\nu_{50:10} = 25000 \times A_{50:10} - 710.5285 \times \ddot{a}_{50:10}
\]

\[
= 25000 \times (A_{50} - 10E_{50}A_{60} + 10E_{50}) - 710.5285 \times (\ddot{a}_{50} - 10E_{50} \ddot{a}_{60})
\]

\[
= 25000 \times (0.24905 - 0.51081 \times 0.36913 + 0.51081) - 710.5285 \times (13.2668 - 0.51081 \times 11.1454)
\]

\[
= 8901.3462
\]
3. A fully discrete whole life insurance policy on (70) has a death benefit of 70,000 and annual premiums for the life of the insured.

You are given that mortality follows the Illustrative Life Table and \( i = 0.06 \).

Calculate \( p^{FPT}_1 \), the first year net premium under Full Preliminary Term reserves and \( 12V^{FPT} \), the reserve at the end of the 12th year under Full Preliminary Term reserves.

\[
p^{FPT}_1 = \sum_{x=0}^{\infty} q_x = 70000 \times \frac{1}{1.06} \times 0.03318 = 2191.132075
\]

\[
p^{FPT}_x = \frac{70000 \cdot A_{x+i}}{\bar{a}_{x+i+1}} = \frac{70000 \times 0.53026}{8.2988} = 4472.718947
\]

\[
12V^{FPT} = 70000 \cdot A_{82} - 4472.718947 \times \bar{a}_{82} = 24397.73956
\]
4. For a fully discrete whole life policy to (65) with a death benefit of 75,000, the annual gross premium is 4000.

You are given the following expenses are used to determine the reserves:

i. Commissions are 40% of premiums in the first year and 5% thereafter.

ii. Per policy expenses are 40 at the beginning of every year including the first year.

You are also given:

i. \( t_{10}V = 13,000 \)

ii. \( i = 0.04 \)

iii. \( q_{75} = 0.050 \)

iv. \( q_{76} = 0.057 \)

Calculate \( 10.3V^g \).

\[
\begin{align*}
11V^g &= \frac{13000 + 4000 - 0.05 \times 4000 - 40 \times (1 + 0.04)}{1 - 0.05} - 75000 \times 0.05 \\
&= 14400.42105
\end{align*}
\]

\[
\begin{align*}
10.3V^g &= (1 - 0.3) \times (11V^g + 0.05P + 40) + 0.3 \times 11V^g \\
&= 0.7 \times (13000 + 0.95 \times 4000 - 40) + 0.3 \times 14400.42105 \\
&= 16052.12632
\end{align*}
\]
6. A fully discrete whole life insurance policy to (45) has a death benefit of 200,000 and annual premiums for as long as the insured is alive.

You are given the following expenses were used for the reserves and the gross premium:
   i. Commissions of 50% of premiums the first year and 8% of premiums in renewal years.
   ii. Issue expenses at the beginning of the first year of 150 per policy.
   iii. Maintenance expenses of 25 per policy at the start of each year including the first year.

You are also given that mortality follows the Illustrative Life Table and \( i = 0.06 \).

The net benefit premium is 2851.45 and the gross premium is 3243.05.

Calculate the expense reserve at the end of the 5th year.

\[
\begin{align*}
5V^h &= 200000 \times A_{50} - 2851.45 \times a_{50} \\
&= 200000 \times 0.24905 - 2851.45 \times 13.2668 \\
&= 11980.38314
\end{align*}
\]

\[
\begin{align*}
5V^g &= 200000 \times A_{50} + (0.08 \times 3243.05 + 25) \times a_{50} - 3243.05 \times a_{50} \\
&= 200000 \times 0.24905 + (25 - 0.02 \times 3243.05) \times 13.2668 \\
&= 10558.76592
\end{align*}
\]

\[
5V^e = 5V^g - 5V^h = -1421.61722
\]
The part highlighted in yellow is repeated from Question 6.

7. A fully discrete whole life insurance policy to (45) has a death benefit of 200,000 and annual premiums for as long as the insured is alive.

You are given the following expenses were used for the reserves and the gross premium:

i. Commissions of 50% of premiums the first year and 8% of premiums in renewal years.

ii. Issue expenses at the beginning of the first year of 150 per policy.

iii. Maintenance expenses of 25 per policy at the start of each year including the first year.

You are also given that mortality follows the Illustrative Life Table and \( i = 0.06 \).

The net benefit premium is 2851.45 and the gross premium is 3243.05.

The actual experience during the 11th year for this policy was:

i. Interest was earned at a rate of 5.75%

ii. Expenses were 7% of premium and 30 per policy at the start of the year.

iii. Mortality was actually 90% of the Illustrative Life Table

You calculate the profit by source allocating the profit first to interest, then to expenses, and finally to mortality.

Calculate the gain from expenses in the 11th year.

\[
\begin{align*}
10V &= 200000 \times A_{55} + (0.08 \times 3243.05 + 25) \times A_{55} - 3243.05 \times A_{55} \\
&= 200000 \times 0.30514 + (25 - 0.92 \times 3243.05) \times 12.758 = 2470.87447 \\
nV &= \frac{(2470.87447 + 3243.05 \times (1 - 0.07) - 25) \times 1.06 - 200000 \times 0.00896}{1 - 0.00896} \\
&= 2774.33918 \\
nV^A &= \frac{(2470.87447 + 3243.05 \times (1 - 0.07) - 30) \times 1.0575 - 200000 \times 0.00896 \times 0.9}{1 - 0.00896 \times 0.9} \\
&= 27899.41173 \\
\text{Profit} &= nV^A - nV = 27899.41173 - 2774.33918 = 115.07255 \\
\text{Due to Interest} &= \frac{1.06 (2470.87447 + 3243.05 \times 0.92 - 25) \times 1.0575 - 200000 \times 0.00896}{1 - 0.00896} = 27714.54545 \\
\text{Due to Expense} &= \frac{(2470.87447 + 3243.05 \times 0.93 - 30) \times 1.0575 - 200000 \times 0.00896}{1 - 0.00896} = 27714.80869 \\
\end{align*}
\]
8. For an insurance policy, the insured can be in one of three states:
   i. State 0 is healthy
   ii. State 1 is disabled
   iii. State 2 is dead

A person can transition from State 0 to State 1 or State 2. Additionally, a person in State 1 can transition into State 2. Finally, a person in State 2 cannot transition.

You are given:
   i. $\mu_{x}^{01} = 0.04$
   ii. $\mu_{x}^{02} = 0.02$
   iii. $\mu_{x}^{22} = 0.05$

Calculate $gP_{x}^{02}$.

\[
gP_{x}^{02} = 1 - gP_{x}^{00} - gP_{x}^{01}
\]

\[
gP_{x}^{02} = \exp \left( - \int_{0}^{\infty} (\mu_{x+1}^{01} + \mu_{x+1}^{02}) \, dt \right) = e^{-0.06 \times 8} = e^{-0.48}
\]

\[
gP_{x}^{01} = \int_{0}^{\infty} gP_{x}^{00} \mu_{x+1}^{01} gP_{x+1}^{00} \, dt
\]

\[
= \int_{0}^{8} e^{-0.06 t} \cdot 0.04 \cdot e^{-0.25 (8-t)} \, dt
\]

\[
= \int_{0}^{8} e^{-0.06 t - 0.4 + 0.25 t} \cdot 0.04 \, dt = 0.04 e^{-0.4} \int_{0}^{8} e^{-0.01 t} \, dt
\]

\[
= 0.04 \cdot e^{-0.4} \cdot \frac{e^{-0.21 t}}{-0.21} \bigg|_{0}^{8} = -4e^{-0.4} \left( e^{-0.21 \cdot 8} - e^{0} \right) = 0.20615
\]

\[
gP_{x}^{00} = 1 - e^{-0.4} \cdot 0.206146617 = 0.17507
\]
9. For an insurance policy, the insured can be in one of three states:
   i. State 0 is healthy
   ii. State 1 is disabled
   iii. State 2 is dead

A person can transition from State 0 to State 1 or State 2. Additionally, a person in State 1 can transition into State 0 or State 2. However, a person in State 2 cannot transition.

You are given the following forces of transition:

i. \( \mu_{x}^{01} = 0.06 \)
ii. \( \mu_{x}^{02} = 0.03 + 0.012t \)
iii. \( \mu_{x}^{12} = (0.12)(1.1^t) \)
iv. Only one transition may occur during any one month period.

Using the Euler method, calculate 1000 times the probability that a person who is healthy today is disabled at the end of 2 months.

\[
\begin{bmatrix}
-0.06x_{0.5} & 0.06x_{0.5} & (0.03 + 0.012x_{0.5})x_{0.5} \\
0.03x_{0.5} & 1 - 0.05x_{0.5} & (0.12x_{1.1})^{1/12} \\
0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.9724167 & 0.005 & 0.032 \\
0.05 & 0.98563 & 0.01 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.99233 & 0.005 & 0.032 \\
0.05 & 0.99567 & 0.01016 \\
0.05 & 0.99567 & 0.01016
\end{bmatrix}
\]

Time 0 1 2
0 0 0 1
0 1 1 1

\[
\begin{align*}
P_{1} &= (0.004962 + 0.005 	imes 0.92835) 	imes 1000 = 9.894335
\end{align*}
\]
10. Actuarial students can be in one of three states:
   i. State 0 is actively taking exams
   ii. State 1 is taking a break from exams
   iii. State 2 is no longer taking exams (either because they quit or because they are finished with exams)

A student can transition from State 0 to State 1 or State 2. Additionally, a student in State 1 can transition into State 0 or State 2. However, a student in State 2 cannot transition.

The following matrix represents the probability of transition between any two states in a year:

\[
\begin{bmatrix}
0.7 & 0.2 & 0.1 \\
0.4 & 0.1 & 0.5 \\
0 & 0 & 1
\end{bmatrix}
\]

Today, Akey Assurance Company has 70 actuarial students who are actively taking exams and 30 actuarial students who are taking a break.

\( N \) is the number of these 100 students who will still be actively taking exams at the end of 2 years.

Calculate \( E[N] \).

\[
\begin{align*}
70 \times 0.7 + 30 \times 0.4 & = 61 \\
61 \times 0.7 + 17 \times 0.4 & = 49.5 \\
70 \times 0.2 + 30 \times 0.1 & = 17 \\
61 \times 0.2 + 17 \times 0.1 & = 13.9 \\
70 \times 0.1 + 30 \times 0.5 & = 22 \\
61 \times 0.1 + 17 \times 0.5 + 22 & = 36.6
\end{align*}
\]

\( E[N] = 49.5 \)
11. For an insurance policy, the insured can be in one of three states:
   i. State 0 is healthy
   ii. State 1 is in a nursing home
   iii. State 2 is dead

A person can transition from State 0 to State 1 or State 2. Additionally, a person in State 1 can transition into State 0 or State 2. However, a person in State 2 cannot transition.

Ahmad Assurance Company issues a three year term insurance policy that pays 50,000 at the end of the year of death. It also pays 100,000 at the end of each year that a person is in a nursing home. The insured pays annual premiums each year during the policy provided the insured is healthy.

You are given that \( \nu = 0.9 \) and the following transitional probability matrix:

\[
\begin{bmatrix}
0.70 & 0.20 & 0.10 \\
0.35 & 0.15 & 0.50 \\
0 & 0 & 1
\end{bmatrix}
\]

Calculate the net benefit premium for this insurance policy assuming that the policy is only sold to a healthy person.

\[
P = \frac{1652445 + 4179375}{20836} = 27989.15339
\]