1. You are given:
   
   i. Mortality follows the illustrative life table
   
   ii. \( i = 6\% \)

   Calculate:

   a. The actuarial present value for a whole life insurance with a death benefit of 100,000 payable at the end of the year of death of (70).

   b. The actuarial present value for a 20 year term insurance with a death benefit of 100,000 payable at the end of the year of death of (70).

   c. The actuarial present value for a 20 year endowment insurance with a death benefit of 100,000 payable at the end of the year of death of (70).

   The actuarial present value of the term insurance is less than the actuarial present value of the whole life which is less than the actuarial present value of the endowment insurance. This will always be true for the same \( x \) and \( \bar{n} \). Explain why.
2. You are given the following mortality table:

<table>
<thead>
<tr>
<th>[x]</th>
<th>( q_x )</th>
<th>( q_{x+1} )</th>
<th>( q_{x+2} )</th>
<th>( q_{x+3} )</th>
<th>( x+3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.020</td>
<td>0.031</td>
<td>0.043</td>
<td>0.056</td>
<td>53</td>
</tr>
<tr>
<td>51</td>
<td>0.025</td>
<td>0.037</td>
<td>0.050</td>
<td>0.065</td>
<td>54</td>
</tr>
<tr>
<td>52</td>
<td>0.030</td>
<td>0.043</td>
<td>0.057</td>
<td>0.072</td>
<td>55</td>
</tr>
<tr>
<td>53</td>
<td>0.035</td>
<td>0.049</td>
<td>0.065</td>
<td>0.091</td>
<td>56</td>
</tr>
<tr>
<td>54</td>
<td>0.040</td>
<td>0.055</td>
<td>0.076</td>
<td>0.113</td>
<td>57</td>
</tr>
<tr>
<td>55</td>
<td>0.045</td>
<td>0.061</td>
<td>0.090</td>
<td>0.140</td>
<td>58</td>
</tr>
</tbody>
</table>

You are also given that \( d = 0.08 \).

\( Z \) is the present value random variable for a 3 year endowment insurance with a death benefit of 1000 payable at the end of the year of death for a person who is age 53 and was underwritten at age 53.

Calculate the variance of \( Z \).
3. You are given:

i. \( v = 0.94 \)
ii. \( E_x = 0.91650000 \)
iii. \( E_y = 0.82532658 \)
iv. \( A_x = 0.500 \)

Calculate \( A_y \)
4. The following information is from the 2000 US Population Table:

<table>
<thead>
<tr>
<th>Age $x$</th>
<th>$q_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>0.4</td>
</tr>
<tr>
<td>111</td>
<td>0.6</td>
</tr>
<tr>
<td>112</td>
<td>0.8</td>
</tr>
<tr>
<td>113</td>
<td>1</td>
</tr>
</tbody>
</table>

Calculate $e_{110}$. 

Is a Population Table appropriate to use to price a life insurance product? Explain your answer!
5. You are given that mortality follows Gompertz law with $B = 0.00025$ and $c = 1.05$.

You are also given that
\[
\int_0^\infty (1.05)^{-t} \cdot \exp \left\{ - \frac{0.00025}{0.048790164} (1.05^t - 1) \right\} dt = 20
\]

Calculate $10,000\bar{A}_0$ using $i = 10.25\%$. Note that $(1.05)^2 = 1.1025$. 
6. You are given that mortality follows the Illustrative Life Table.

\[ 2.5q^{CFM}_{80.2} \] is calculated assuming that a constant force of mortality applies between integral ages.

\[ 2.5q^{UDD}_{80.2} \] is calculated assuming that deaths are uniformly distributed between integral ages.

Calculate \[ 10,000\left(2.5q^{CFM}_{80.2} - 2.5q^{UDD}_{80.2}\right) \].

Your boss wants to know which assumption (UDD or CFM) you would recommend and why. Be sure to explain why.
7. You are given that $\mu_{x,t} = 0.01t + 0.002t^2$.

Calculate $10^x P_x$. 
8. Amstutz Assurance Company provides warranty protection on Christmas tree lights. Under the warranty coverage, if a stand of lights burns out during the year, Amstutz will replace the strand of lights at end of the year. The coverage provided has a three year term and coverage continues on the replaced lights. In other words, if the strands that replace the burned out strands also stop working, Amstutz will replace them also.

The city of West Lafayette purchases coverage and has 50,000 strands of Christmas lights which are all new.

Christmas light strands burn out at the following rates:

i. The probability that a new strand of lights will fail in the first year of service is 20%.
ii. The probability that a one year old strand of lights will fail during the next year is 45%.
iii. The probability that a two year old strand of lights will fail during the next year is 60%.

A cost to replace a strand of lights is 2.

Using an interest rate of 8%, calculate the Actuarial Present Value of the coverage offered by Amstutz to West Lafayette.
9. You are given:

i. $10 q_{58} = 0.10$

ii. $20 q_{58} = 0.25$

iii. $30 q_{58} = 0.50$

Calculate $20 P_{68}$