1. A whole life insurance policy on (70) provides a death benefit of 150,000 payable at the end of the year of death. The premiums are paid annually for the life of the insured and are calculated using the equivalence principle.

You are given:

i. Mortality follows the Illustrative Life Table.
ii. $i = 0.06$
iii. The policy incurs the following expenses:
   1. Commissions of 55% of premiums in the first year and 6% of premiums thereafter
   2. Per policy issue expenses of 125
   3. Per policy maintenance expenses of 25 at the start of every year including the first year.

a. (3 points) The net benefit premium is 9000 to the nearest 100. Calculate the net benefit premium to the nearest 0.01.

b. (8 points) Calculate $9.3V^n$. 


c. \textit{(3 points)} The gross premium is 10,300 to the nearest 100. Calculate the gross premium to the nearest 0.01.

d. \textit{(5 points)} Calculate the gross premium reserve at the end of the 10\textsuperscript{th} policy year.

e. \textit{(3 points)} Calculate the expense reserve at the end of the 10\textsuperscript{th} policy year.
f. (7 points) Let $L_0^g$ be the loss at issue random variable based on the gross premium.

$L_0^g$ can be written as $A v^{K_{w+1}} + B + C \bar{u}_{K_{w+1}}$.

Determine $A$, $B$, and $C$.

g. (7 points) Determine the standard deviation of $L_0^g$. 
2. A 10 year term insurance policy on (50) pays a death benefit of 1,000,000 at the moment of death. The policy has annual premiums payable for 10 years while the insured is alive. Reserves are determined using the Full Preliminary Term net premium reserve method.

You are given:
   i. Mortality follows the Illustrative Life Table.
   ii. $i = 0.06$

Let $P^1$ be the first year net premium under Full Preliminary Term and $P^2$ be the net premium in years two and later.

$P^2 - P^1$ is known as the expense allowance.

a. **(8 points)** Calculate the expense allowance for this policy.

b. **(6 points)** Calculate the Full Preliminary Term reserve at the end of 5 years.
3. \textit{(8 points)} A whole life insurance policy on (50) pays a death benefit of 100,000 at the end of the year of death. The policy has annual gross premiums of 2250 payable for as long as the insured is alive.

During the first two years, the policy has the following experience:

i. Mortality follows the Illustrative Life Table
ii. $i = 0.07$
iii. Issue expenses were 350 per policy.
iv. Maintenance expense was 50 per policy at the beginning of both years.
v. Commissions were 60\% of premium in the first year and 8\% of premium in the second year.

Calculate the asset share at the end of the second year.
4. A long term care policy is modeled using a multi-state model. The model has three states:

   i. State 0 is Healthy
   ii. State 1 is in a Long Term Care facility
   iii. State 2 is dead

State 0 can transition to State 1 or State 2. State 1 can transition to State 0 or State 2. State 2 cannot transition.

You are given the following forces of transition:

   i. $\mu_{x+t}^{01} = 0.10 + 0.06t$
   ii. $\mu_{x+t}^{10} = 0.18$
   iii. $\mu_{x+t}^{02} = 0.03(1 + t)$
   iv. $\mu_{x+t}^{12} = 0.3 + 0.12t$

a. **(6 points)** Using the Euler method with $h=1/12$, $2/12 \, p^{11}_{x+t}$. 
A Nursing Home has 1000 residents who were age \( x \). Each resident’s future state is independent of the future state of any other resident. Let \( N \) be the number of residents of these 1000 that will still be in the Nursing home after 2 months.

b. (3 points) Calculate \( E[N] \) and \( Var[N] \)
5. Lives insured can be modeled using a multi-state model. The model has three states:

i. State 0 is Healthy
ii. State 1 is Disabled
iii. State 2 is Dead

State 0 can transition to State 1 or State 2. State 1 can transition to State 0 or State 2. State 2 cannot transition.

You are given the following matrix of annual transition probabilities.

\[
\begin{bmatrix}
0.80 & 0.15 & 0.05 \\
0.30 & 0.60 & 0.10 \\
0 & 0 & 1
\end{bmatrix}
\]

An insurance company decides to issue a three year term insurance policy which pay a benefit of 10,000 at the end of the year of death. The policy will also pay 50,000 at the end of each year that the insured is disabled. Annual premiums will be paid only by healthy lives. The premium is determined using the equivalence principle.

a. (6 points) If \( v = 0.95 \), calculate the annual premium.
Let $V$ be the average reserve for all insureds who are alive at time $t$. Let $V^{(0)}$ be the reserve for insureds who are in state 0 at time $t$. Let $V^{(1)}$ be the reserve for insureds who are in state 1 at time $t$.

b. (2 points) Calculate $V$.

c. (2 points) Calculate $V^{(0)}$.

d. (3 points) Calculate $V^{(1)}$. 
6. (8 points) Actuarial science students can be modeled using a multi-state model. The model has three states:
   i. State 0 is actively taking exams
   ii. State 1 is not actively taking exams but could resume taking exams
   iii. State 2 is will never take another exam (may be done with exams, given up on exams, or are dead)

   State 0 can transition to State 1 or State 2. State 1 can transition to State 2. State 2 cannot transition.

   You are given the following force of transitions:
   
   i. \( \mu_{x+1}^{01} = 0.3 \)
   ii. \( \mu_{x+1}^{02} = 0.1 \)
   iii. \( \mu_{x+1}^{12} = 0.6 \)

   Calculate \( sP_x^{02} \). 

7. A 30 year endowment policy issued on (60) pays a death benefit of 70,000 at the end of the year of death. The annual gross premium paid during the life of the contract is 2813.02.

You are given:

i. Mortality follows the Illustrative Life Table
ii. \( i = 0.06 \)
iii. The gross premium reserves are based on the following assumptions:
   1. Commission are 80% of premiums in the first year and 8% thereafter
   2. Per policy issue expense of 190
   3. Annual maintenance expense of 25 per policy at the beginning of each year after the first year.
iv. \( 10V^g = 11,779.59 \)

Actual experience in the 11\(^{th}\) policy year is:

i. Mortality is 110% of the Illustrative Life Table
ii. \( i = 0.07 \)
iii. The expenses are:
   1. 7\% of premium for commissions
   2. Annual maintenance expense of 20 per policy at the beginning of the year

a. \( \text{(6 points) Calculate the total profit in the 11\(^{th}\) year.} \)
b. *(6 points)* If the company allocates the profit by source to expenses first, then interest, and finally mortality, determine the profit allocated to interest.