1. You are given:
   a. \( q_{80} = 0.08 \)
   b. \( q_{81} = 0.10 \)
   c. \( q_{82} = 0.12 \)

Deaths are uniformly distributed between ages 80 and 81. Deaths are subject to a constant force of mortality between ages 81 and 82 as well as between 82 and 83.

Calculate \( 0.7|1.2\ 80.4 \). 

\[
0.7 \| 1.2 q_{80.4} = \frac{l_{81.1} - l_{82.3}}{l_{80.4}}
\]

CFM : \( l_x + y = l_x (1-x) l_x + 1 (x) \)

UDD : \( l_x + y = l_x (1-s) + l_x + 1 (s) \)

\( l_{80} = 1000 \)

\( l_{81} = l_{80} * p_{80} = 1000 * 0.92 = 920 \)

\( l_{82} = l_{81} * p_{81} = 920 * 0.9 = 828 \)

\( l_{83} = l_{82} * p_{82} = 828 * 0.88 = 728.64 \)

\[
\frac{l_{81.1} - l_{82.3}}{l_{80.4}} = \frac{920^{0.9} * 828^{0.1} - 828^{0.7} * 728.64^{0.3}}{0.6 * 1000 + 0.4 * 920}
\]

\[
\frac{l_{81.1} - l_{82.3}}{l_{80.4}} = \frac{910.3577 - 796.8474}{968}
\]

\[
\frac{l_{81.1} - l_{82.3}}{l_{80.4}} = 0.11726
\]
2. You are given the following select and ultimate mortality table which reflects mortality today:

<table>
<thead>
<tr>
<th>x</th>
<th>q_{x}</th>
<th>q_{x+1}</th>
<th>q_{x+2}</th>
<th>x+2</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>0.05</td>
<td>0.08</td>
<td>0.14</td>
<td>77</td>
</tr>
<tr>
<td>76</td>
<td>0.07</td>
<td>0.12</td>
<td>0.18</td>
<td>78</td>
</tr>
<tr>
<td>77</td>
<td>0.10</td>
<td>0.15</td>
<td>0.25</td>
<td>79</td>
</tr>
</tbody>
</table>

Mortality is expected to improve at a rate of 1.25% per year into the future.

Calculate \( q'_{75+1} \).

\[
q' = q_{\text{reduced}}
\]

\[
q'_{75} + 1 = 0.08
\]

\[
q'_{75} + 2 = 0.14(1 - 0.0125) = 0.13825
\]

\[
q'_{75 + 3} = 0.18(1 - 0.0125)^2 = 0.17553
\]

\[
p'_{75} + 1 = 1 - q'_{75} + 1 = 0.92
\]

\[
p'_{75} + 2 = 1 - q'_{75} + 2 = 0.86175
\]

\[
p'_{75 + 3} = 1 - q'_{75 + 3} = 0.82447
\]

\[
3q'_{75} + 1 = 1 - 3p'_{75} + 1
\]

\[
3q'_{75} + 1 = 1 - p'_{75} + 1 * p'_{75} + 2 * p'_{75 + 3}
\]

\[
3q'_{75} + 1 = 0.34635
\]
1. You are given the following select and ultimate mortality table which reflects mortality today:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$q_x$</th>
<th>$q_{x+1}$</th>
<th>$q_{x+2}$</th>
<th>$x+2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>0.05</td>
<td>0.08</td>
<td>0.14</td>
<td>77</td>
</tr>
<tr>
<td>76</td>
<td>0.07</td>
<td>0.12</td>
<td>0.18</td>
<td>78</td>
</tr>
<tr>
<td>77</td>
<td>0.10</td>
<td>0.15</td>
<td>0.25</td>
<td>79</td>
</tr>
</tbody>
</table>

Deaths are subject to a constant force of mortality between ages [75] and [75]+1. Deaths are uniformly distributed between ages [75]+1 and 77 as well as between 77 and 78.

Calculate $1.1|0.5 q_{[75]+0.6}$

$$1.1 | 0.5 q_{[75]+0.6} = \frac{l_{[75]} + 1.7 - l_{[75]} + 2.2}{l_{[75]} + 0.6}$$

**CFM:** $l_{x+s} = l_x (1-s) + l_{x+1}(s)$

**UDD:** $l_{x+s} = l_x (1-s) + l_{x+1}(s)$

$l_{[75]} = 1000$

$l_{[75]} + 1 = l_{[75]} * p_{[75]} = 1000 * 0.95 = 950$

$l_{[75]} + 2 = l_{[75]} + 1 * p_{[75]} + 1 = 950 * 0.92 = 874$

$l_{[75]} + 3 = l_{[75]} + 2 * p_{[75]} + 2 = 874 * 0.86 = 751.64$

$$\frac{l_{[75]} + 1.7 - l_{[75]} + 2.2}{l_{[75]} + 0.6} = \frac{0.3*950 + 0.7*874 - 0.8*874 + 0.2*751.64}{1000^{0.4} * 950^{0.6}} = 0.3*950 + 0.7*874 - 0.8*874 + 0.2*751.64$$

$$\frac{l_{[75]} + 1.7 - l_{[75]} + 2.2}{l_{[75]} + 0.6} = \frac{896.8 - 849.528}{96.969} = 0.04875$$

$0.4875$
2. You are given:

   a. \( q_{80} = 0.04 \) for \( t = 0,1,\ldots,9 \) and
   
   b. \( q_{80} = 0.06 \) for \( t = 10,11,\ldots,19 \)

There are 1000 lives age 85 subject to this mortality. \( L_{10} \) is the random variable representing the number that will still be alive at age 95.

Calculate \( E[L_{10}] \) and \( Var[L_{10}] \)

\[
\begin{align*}
0 | 1 q_{85} = q_{85} &= 0.04 = \frac{l_{80} - l_{81}}{l_{80}} \\
l_{80} &= 100 \\
0.04 &= \frac{100 - l_{81}}{100} \rightarrow l_{81} = 96 \\
l_{82} &= 96 - 4 = 92 \\
l_{85} &= 100 - (4)(5) = 80 \\
l_{90} &= 100 - (4)(10) = 60 \\
l_{91} &= 60 - 6 = 54 \\
l_{92} &= 60 - 6(2) = 48 \\
l_{95} &= 60 - 6(5) = 30 \\
_{10} p_{85} &= \frac{l_{95}}{l_{85}} = \frac{30}{80} = \frac{3}{8} \\
L_{10} &\sim Bin(1000, _{10} p_{85}) \\
Bin : E[L_{10}] &= np = 1000 * _{10} p_{85} \\
Bin : Var[L_{10}] &= npq = 1000 * _{10} p_{85} * _{10} q_{85} \\
E[L_{10}] &= 1000 * \frac{3}{8} = 375 \\
Var[L_{10}] &= 1000 * \frac{3}{8} * \frac{5}{8} = 234.375
\end{align*}
\]