1. (20 points) $Y$ is the present value random variable for a whole life annuity due to (x) with annual payments of 1.

You are given that:

i. $1000A_x = 700$

ii. $1000\left[2A_x\right] = 530$

iii. $\text{Var}[Y] = 4$

iv. Deaths are uniformly distributed between integer ages.

Calculate $\bar{a}_x$.

**Solutions:**

$$\text{Var}[Y] = \frac{2A_x - (A_x)^2}{d^2} = 4 \implies \frac{0.530 - (0.7)^2}{d^2} = 4 \implies d = \sqrt{\frac{0.4 - 0.04}{4}} = 0.1$$

$$i = \frac{d}{1 - d} = \frac{0.1}{0.9} = \frac{1}{9}$$

$$\bar{a}_x = \frac{1 - \bar{A}_x}{\delta} = \frac{1 - \frac{i}{\delta}A_x}{\delta} = \frac{1 - \frac{1/9}{\ln(1+1/9)}(0.7)}{\ln(1+1/9)} = 2.4847$$
2. (10 points) I would like to receive 10 points for putting my name on the cover of this quiz. (Circle the correct answer.)

True or False
1. (20 points) A life annuity due payable to (70) pays annual payments of 1000.

You are given:

i. Mortality follows the Illustrative Life Table.
ii. \( i = 6\% \)
iii. \( Y \) is the present value random variable for this annuity.

Calculate the probability that \( Y \) will be greater than the expected value of \( Y \) plus the standard deviation of \( Y \).

Solution:

\[
E[Y] = 1000 \ddot{a}_{70} = 1000(8.5693) = 8569.30
\]

\[
Var[Y] = (1000)^2 \left[ \frac{2A_{70} - (A_{70})^2}{d^2} \right] = (1000)^2 \left[ \frac{0.30642 - (0.51495)^2}{(0.05660)^2} \right] = (1000)^2(12.8752)
\]

\[
SD = \sqrt{(1000)^2(12.8752)} = 3588.20
\]

\[
Pr[1000Y > 8569.30 + 3588.20] = Pr[Y > 12.1575] = Pr\left[ \frac{1 - v^{K+1}}{d} > 12.1575 \right] =
\]

\[
Pr[v^{K+1} < 1 - 12.1575(d)] = Pr\left[ K_x > \frac{\ln(1 - 12.1575(0.05660(1 - 0.1936)))}{\ln(1.06^{-1})} - 1 \right] =
\]

\[
Pr[K_x > 18.99] = \frac{l_{89}}{l_{70}} = \frac{1,281,083}{6,616,155} = 0.1936
\]
2. (10 points) I would like to receive 10 points for putting my name on the cover of this quiz. (Circle the correct answer.)

True or False