1. You are given that mortality follows the Illustrative Life Table except for age 70 and 71. For ages 70 and 71, $1000q_{70} = 25$ and $1000q_{71} = 30$.

You are also given that $i = 6\%$.

Calculate $\ddot{a}_{70}$.

**Solution:**

$$\ddot{a}_x = 1 + vp_x \cdot \ddot{a}_{x+1}$$

$$\ddot{a}_{71} = 1 + vp_{71} \cdot \ddot{a}_{72} \implies \ddot{a}_{71} = 1 + \frac{1 - 0.03}{1.06} \cdot 8.0278 = 8.34619$$

$$\ddot{a}_{70} = 1 + vp_{70} \cdot \ddot{a}_{71} \implies \ddot{a}_{70} = 1 + \frac{1 - 0.025}{1.06} \cdot 8.34619 = 8.676924$$
2. You are given that mortality follows the Illustrative Life Table and $i = 6\%$. You are also given that deaths are uniformly distributed between integral ages.

Calculate $300\hat{a}_{50:10}^{(4)}$.

**Solution:**

$$300\hat{a}_{50:10}^{(4)} = 300\left[\hat{a}_{10}^{(4)} + E_{50} \cdot \hat{a}_{60}^{(4)}\right] = 300 \left[\frac{1 - 1.06^{-10}}{d^{(4)}} + E_{50} \cdot \left\{\alpha(4) \cdot \hat{a}_{60} - \beta(4)\right\}\right] =$$

$$300 \left[\frac{1 - 1.06^{-10}}{0.05785} + (0.51081) \cdot \{(1.00027)(11.1454) - 0.38424\}\right] = 3939.62$$
1. You are given that mortality follows the Illustrative Life Table and \( i = 6\% \).

Calculate \( 300\dot{a}_{\overline{80}}^{(12)} \) using the three factor Woolhouse formula.

**Solution:**

\[
300\dot{a}_{\overline{80}}^{(12)} = 300 \left[ \hat{a}_{\overline{80}} - \frac{12 - 1}{2(12)} - \frac{12^2 - 1}{12(12)^2} (\mu_x + \delta) \right] = \\
300 \left[ 5.9050 - \frac{11}{24} - \frac{143}{1728} \left\{ \frac{\ln p_{79} + \ln p_{80}}{2} + \ln(1.06) \right\} \right] = \\
300 \left[ 5.9050 - \frac{11}{24} - \frac{143}{1728} \left\{ \frac{\ln(1 - 0.07356) + \ln(1 - 0.08030)}{2} + \ln(1.06) \right\} \right] = \\
= 1630.57
\]
2. You are given that mortality follows the Illustrative Life Table and \( i = 6\% \).

A special annuity due makes annual payments for the life of (40). The first 10 payments are 1000. The second 10 payments are 2000. All payments thereafter are 1500.

Calculate the actuarial present value of this annuity.

Solution:

\[
APV = 1000\ddot{a}_{40} + 1000\ddot{a}_{40}E_{40}\ddot{a}_{50} - 500\ddot{a}_{40}E_{40}\ddot{a}_{50} = \\
1000(14.8166) + 1000(0.53667)(13.2668) - 500(0.27414)(11.1454) = 20,408.79
\]