1. (20 points) For an insurance policy, the insured can be in one of three states:
   i. State 0 is healthy
   ii. State 1 is disabled
   iii. State 2 is dead

   A person can transition from State 0 to State 1 or State 2. Additionally, a person in State 1 can transition into State 2. Finally, a person in State 2 cannot transition.

   You are given:
   i. \( \mu_{x}^{01} = 0.03 \)
   ii. \( \mu_{x}^{02} = 0.01 \)
   iii. \( \mu_{x}^{12} = 0.04 \)

   Calculate \( 12 P_{x}^{02} \).

   Solution:

   \[
   \int_{0}^{t} e^{-0.04t} = e^{-0.04t} = e^{-0.04t} \Rightarrow 12 P_{x}^{00} = e^{-0.04t} = 0.61878
   \]

   \[
   12 P_{x}^{01} = \int_{0}^{12} P_{x}^{00} \cdot \mu_{x+1}^{01} \cdot P_{x+1}^{11} dt = \int_{0}^{12} e^{-0.04t} \cdot 0.03 \cdot e^{-0.04t} dt = \int_{0}^{12} e^{-0.04t} \cdot 0.03 \cdot e^{-0.04t} dt = 0.61878 - 0.22276
   \]

   \[
   0.03 \cdot e^{-0.48} \int_{0}^{12} e^{0} dt = 0.03 \cdot e^{-0.48} \int_{0}^{12} e^{0} dt = 0.03 \cdot e^{-0.48} \left[ t \right]_{0}^{12} = 0.03 \cdot e^{-0.48} (12) = 0.22276
   \]

   \[
   12 P_{x}^{02} = 1 - 12 P_{x}^{00} - 12 P_{x}^{01} = 1 - 0.61878 - 0.22276 = 0.15846
   \]
2. (20 points) For an insurance policy, the insured can be in one of three states:

iv. State 0 is healthy
v. State 1 is disabled
vi. State 2 is dead

A person can transition from State 0 to State 1 or State 2. Additionally, a person in State 1 can transition into State 2. Finally, a person in State 2 cannot transition.

You are given:

iv. \( \mu_{x}^{01} = 0.025 \)

v. \( \mu_{x}^{02} = 0.015 \)

vi. \( \mu_{x}^{12} = 0.06 \)

Calculate \( P_x^{02} \).

Solution:

\[
P_x^{00} = e^{-\int_{0}^{t} (0.05 \cdot t + 0.15) dt} = e^{-0.04t} = e^{-0.04} \quad \Rightarrow 10 \quad P_x^{00} = e^{-0.4} = 0.6703
\]

\[
10 \quad P_x^{01} = \int_{0}^{10} P_x^{00} \cdot \mu_{x+1}^{01} \cdot 10 \quad P_x^{11} \quad dt = \int_{0}^{10} e^{-0.04r} \cdot 0.025 \cdot e^{-0.06} \quad dt
\]

\[
\int_{0}^{10} e^{-0.04r} \cdot 0.025 \cdot e^{-0.06(10-t)} \quad dt = \int_{0}^{10} e^{-0.04r} \cdot 0.025 \cdot e^{-0.6} \cdot e^{0.06t} \quad dt = 0.025 \cdot e^{-0.6} \int_{0}^{10} e^{0.02t} \quad dt = 0.025 \cdot e^{-0.6} \left[ \frac{e^{0.02t}}{0.02} \right]_{0}^{10} = 0.025 \cdot e^{-0.6} \left( \frac{e^{0.2} - 1}{0.02} \right) = 0.1519
\]

\[
10 \quad P_x^{02} = 1 - 10 \quad P_x^{00} - 10 \quad P_x^{01} = 1 - 0.6703 - 0.1519 = 0.1778
\]