1. (10 points) Wenzhong who is (65) buys a 20 year endowment insurance policy with a death benefit of 82,000 payable at the moment of death.

You are given:

   a. Wenzhong’s mortality follows the Illustrative Life Table.

   b. Deaths are uniformly distributed between integral ages.

   c. \( i = 0.06\% \)

Calculate the expected present value of Wenzhong’s endowment insurance.

\[
EPV = 82,000\bar{A}_{65 : 20} \\
\bar{A}_{65 : 20} = \bar{A}_{65 : 20}^{1} + 20E_{65} \\
\bar{A}_{65 : 20} = \frac{i}{\delta}A_{65 : 20}^{1} + 20E_{65} \\
\bar{A}_{65 : 20} = \frac{i}{\delta}(A_{65} - 20E_{65}A_{85}) + 20E_{65} \\
\bar{A}_{65 : 20} = \frac{0.06}{\ln(1.06)}(0.4398 - 0.0976*0.73407) + 0.0976 \\
\bar{A}_{65 : 20} = 0.47669215 \\
EPV = 82,000\bar{A}_{65 : 20} = 39,088.76
\]
2. (10 points) You are given that today mortality follows the mortality table below:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$q_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>85</td>
<td>0.10</td>
</tr>
<tr>
<td>86</td>
<td>0.20</td>
</tr>
<tr>
<td>87</td>
<td>0.40</td>
</tr>
<tr>
<td>88</td>
<td>0.60</td>
</tr>
<tr>
<td>89</td>
<td>0.80</td>
</tr>
<tr>
<td>90</td>
<td>1.00</td>
</tr>
</tbody>
</table>

You are also given that mortality will improve by 1.5% per year each year in the future.

Calculate $e_{86.3}$.

Reduction factor = 1 - 0.015 = 0.985

<table>
<thead>
<tr>
<th>$x$</th>
<th>$q_x$</th>
<th>$q_x'$</th>
<th>$p_x'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>85</td>
<td>0.10</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>86</td>
<td>0.20</td>
<td>1(0.2) = 0.2</td>
<td>1 - 0.2 = 0.8</td>
</tr>
<tr>
<td>87</td>
<td>0.40</td>
<td>0.985(0.4) = 0.394</td>
<td>1 - 0.394 = 0.606</td>
</tr>
<tr>
<td>88</td>
<td>0.60</td>
<td>0.985²(0.6) = 0.582135</td>
<td>1 - 0.5821 = 0.417865</td>
</tr>
<tr>
<td>89</td>
<td>0.80</td>
<td>0.985³(0.8) = 0.7645373</td>
<td>1 - 0.7645 = 0.2354627</td>
</tr>
<tr>
<td>90</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$\sum_{k=1}^{3} p_{86} = p'_{86} + p'_{86} + p'_{86}$$

$$\sum_{k=1}^{3} p_{86}' = p'_{86} + p'_{86} + p'_{86} + p'_{87} + p'_{87} + p'_{88}$$

$$\sum_{k=1}^{3} p_{86} = 0.8 + 0.8 \times 0.606 + 0.8 \times 0.606 \times 0.417865$$

$$\sum_{k=1}^{3} p_{86} = 1.4874$$
3. (10 points) You are given that mortality follows Makeham’s Law with the following parameters:

1. \( A = 0.004 \)
2. \( B = 0.00003 \)
3. \( c = 1.1 \)

You are also given that \( i = 10\% \).

Let \( L_{15} \) be the random variable representing the number of lives alive at the end 15 years if there are 10,000 lives at age 50. Calculate \( \text{Var}[L_{15}] \)

\[
\text{Var}[L_{15}] = npq = 10,000 \times _{15} p_{50} \times _{15} q_{50}
\]

\[
_{15} p_{50} = e^{-A(15) - \frac{B}{\ln C} \times C^{50} (C^{15} - 1)} = 0.837445
\]

\[
_{15} q_{50} = 1 - _{15} p_{50} = 0.162555
\]

\[
\text{Var}[L_{15}] = 10,000 \times 0.837445 \times 0.162555 = 1361.3
\]
4. (10 points) Tonia purchases a special 30 year term insurance policy. Tonia is (25).

The death benefit for Tonia’s policy changes during the term period. The death benefit is
100,000 if Tonia dies during the first 10 years. The death benefit is 250,000 if Tonia dies after 10
years but before the end of 20 years. Finally, if Tonia dies after 20 years, but before the end of
the 30th year, the death benefit is 500,000. Of course, since it is a term policy, if Tonia lives
longer than 30 years, no death benefit is paid. In all cases, the death benefit is paid at the end
of the year of death.

You are given that the mortality follows the Illustrative Life Table and that \( i = 0.06 \).

Calculate the Expected Present Value of Tonia’s term policy.

\[
PVDB = 100,000A_{25 : 50} + 150,000 * 10 | A_{25 : 50} + 250,000 * 20 | A_{25 : 50}
\]

\[1 = A_{25 : 50} = A_{25 : 30}E_{25} * A_{55} = 0.08165 - 0.30514 * 0.15729 = 0.03365453\]

\[2 = 10 | A_{25 : 50} = 10 | E_{25} * A_{35} - 30 | E_{25} * A_{55} = 0.54997 * 0.12872 - 0.30514 * 0.15729 = 0.0227967\]

\[3 = 20 | A_{25 : 50} = 20 | E_{25} * A_{45} - 30 | E_{25} * A_{55} = 0.29873 * 0.2012 - 0.30514 * 0.15729 = 0.012109\]

\[100,000 * 1 + 150,000 * 2 + 250,000 * 3 = 9,812.20\]

Or

\[
PVDB = 100,000A_{25} + 150,000E_{25} * A_{35} + 250,000E_{25} * A_{45} * 30 | E_{25} * A_{55}
\]

\[= (100,000)(0.08165) + (150,000)(0.54997)(0.12872)\]

\[+ (250,000)(0.29873)(0.2012) - 500,000(0.29873)(0.52652)(0.30514)\]

\[= 9812.61\]
5. The Zhang Life Insurance Company sells a whole life insurance policy with a death benefit of 9000 to insureds who are age 60. The death benefit is paid at the end of the year of death.

The mortality for (60) follows the Illustrative Life Table except for at age 60. Due to underwriting, at age 60, the mortality is 40% of the mortality in the Illustrative Life Table.

You are also given that $i = 0.06$.

a. (10 points) The actuarial present value of the whole life insurance policy is 3275 to the nearest 25. Calculate it to the nearest 1.

Adjusted mortality: $q_{60}' = 0.4 \times q_{60}$

$q_{60}' = 0.4(0.01376) = 0.005504$

$p_{60}' = 1 - q_{60}' = 0.994496$

$A_{60} = vq_{60} + vp_{60} \times A_{61}$

$A_{60} = (1.06)^{-1}(0.005504) + (1.06)^{-1}(0.994496)(0.38279)$

$A_{60} = 0.36433$

$9000A_{60} = 3278.95 \approx 3279$

b. (10 points) $Z$ is the present value random variable for this whole life insurance. Calculate $Var[Z]$.

$Var[Z] = DB^2 \left( 2A_x - \left( A_x \right)^2 \right)$

$Var[Z] = 9000^2 \left( 2A_{60} - \left( A_{60} \right)^2 \right)$

$2A_{60} = v^2 q_{60} + v^2 p_{60} \left( 2A_{61} \right)$

$2A_{60} = (1.06)^{-2}(0.005504) + (1.06)^{-2}(0.994496)(0.18817)$

$2A_{60} = 0.17145$

$Var[Z] = 9000^2(0.17145 - (0.36433)^2) = 3,135,805.74$
Zhang sells 625 policies. Further, Zhang sets aside 2.1 million to cover future death benefits.

c. (10 points) Assuming the normal distribution, calculate the probability that the 2.1 million that Zhang set aside will be LESS than the present value of the benefits actually paid.

\[
E[port] = 625E[Z] = 625(3279) = 2,049,375 \\
Var[port]=625Var[Z]=625(3,135,806)=1,959,878,750 \\
2,100,000=2,049,375+z*\sqrt{Var[port]} \\
50,625 = z*\sqrt{Var[port]} \\
z = 1.1435 \\
Pr[z < 1.14] = 0.8729 \\
Pr[2,100,000 < PV \text{ benefits actually paid}] = 1 - 0.8729 = 0.1271
6. You are given the following select and ultimate table:

| x  | q_{x|x} | q_{x|x+1} | q_{x|x+2} | q_{x+3} | x + 3 |
|----|---------|-----------|-----------|---------|-------|
| 50 | 0.020   | 0.031     | 0.043     | 0.056   | 53    |
| 51 | 0.025   | 0.037     | 0.050     | 0.065   | 54    |
| 52 | 0.030   | 0.043     | 0.057     | 0.072   | 55    |
| 53 | 0.035   | 0.049     | 0.065     | 0.091   | 56    |
| 54 | 0.040   | 0.055     | 0.076     | 0.113   | 57    |
| 55 | 0.045   | 0.061     | 0.090     | 0.140   | 58    |

a. (7 points) Calculate \( 1000(0.6|_{1.5} q_{[52]+1.7}) \) assuming that deaths are uniformly distributed between integral ages.

\[
1000(0.6|_{1.5} q_{[52]+1.7}) = 1000 \frac{l_{[52]+2.3} - l_{[52]+3.8}}{l_{[52]+1.7}}
\]

\[
l_{[52]+2.3} = (0.7)l_{[52]+2} + (0.3)l_{55} = 912.42
\]

\[
l_{[52]+3.8} = l_{55,8} = (0.2)l_{55} + (0.8)l_{56} = 824.96
\]

\[
l_{[52]+1.7} = (0.3)l_{[52]+1} + (0.7)l_{[52]+2} = 940.80
\]

\[
1000(0.6|_{1.5} q_{[52]+1.7}) = 1000 \frac{912.42 - 824.96}{940.80} = 92.96
\]

b. (3 points) When are select and ultimate tables appropriate and explain why.

Select and ultimate tables are appropriate when underwriting takes place, because it adjusts the mortality to give a more accurate and realistic depiction of different individuals’ mortality within a population. Thus, the cost of insurance is more accurate.
7. (10 points) For a new light bulb, you are given that \( q_t = \frac{t^3 + t}{72} \) for \( 0 \leq t \leq 8 \). Let \( T_0 \) be the random variable representing the future lifetime of a new light bulb.

Calculate the \( \text{Var}[T_0] \).

\[
\begin{align*}
\varphi_x &= \int_0^\infty \text{e}^x \cdot t \cdot p_t \, dt \\
E[T_0] &= \int_0^8 \varphi_t \, dt = \int_0^8 (1 - \frac{t^2 + t}{72}) \, dt \rightarrow \frac{1}{72} \left[ 72t - \frac{t^3}{3} - \frac{t^2}{2} \right]_0^8 = 5.1852 \\
E[T_0^2] &= 2 \int_0^8 (t \cdot \varphi_t) \, dt = \frac{2}{72} \int_0^8 (72t^2 + t^3 - t^2) \, dt = \frac{2}{72} \left[ 36t^2 - \frac{t^4}{4} - \frac{t^3}{3} \right]_0^8 = 30.815 \\
\text{Var}[T_0] &= E[T_0^2] - (E[T_0])^2 = 3.9287
\end{align*}
\]
8. (10 points) You are given the following mortality table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$q_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.05</td>
</tr>
<tr>
<td>101</td>
<td>0.15</td>
</tr>
<tr>
<td>102</td>
<td>0.25</td>
</tr>
<tr>
<td>103</td>
<td>0.50</td>
</tr>
<tr>
<td>104</td>
<td>1.00</td>
</tr>
</tbody>
</table>

You are also given that $i = 10\%$.

An increasing whole life policy issued to (101) pays the death benefit at the end of the year of death.

The death benefit in the first year is 1. The death benefit in the second year is 2. The death benefit in the third year is 3, etc.

Calculate the actuarial present value of this increasing whole life insurance.

\[
APV = (1)vq_{101} + (2)v^2p_{101}q_{102} + (3)v^3p_{101}p_{102}q_{103} + (4)v^4p_{101}p_{102}p_{103}q_{104}
\]

Using $d_x$'s:

\[
1000APV = (1)(1.1)^{-1}(150) + (2)(1.1)^{-2}(212.5) + (3)(1.1)^{-3}(318.75) + (4)(1.1)^{-4}(318.75) = 2,076.89
\]

\[APV = 2.07689\]