1. Losses from a policy covering emergency room visits are distributed as a Pareto distribution with $\alpha = 3$ and $\theta = 1000$.

The insurance company wants to impose a deductible such that the expected cost per emergency room visit under the policy is reduced to 50%. In other words:

$$E[(X-d)_+] = 0.5E[X]$$

Determine $d$.

$$E(X) = \frac{\theta}{\alpha-1} = \frac{1000}{3-1} = 500$$

$$E[(X \wedge d)] + E[(X-d)_+] = E[X]$$

$$E[(X \wedge d)] + \frac{1}{2} E[X] = E[X]$$

$$E[(X \wedge d)] = \frac{1}{2} E[X] = \frac{1}{2} (500) = 250$$

$$E[(X \wedge d)] = \frac{\theta}{\alpha-1} \left(1 - \left(\frac{\theta}{\theta+d}\right)^{\alpha-1}\right)$$

$$250 = \frac{1000}{3-1} \left(1 - \left(\frac{1000}{1000+d}\right)^{3-1}\right)$$

$$250 = 500 \left(1 - \left(\frac{1000}{1000+d}\right)^{2}\right)$$

$$\frac{1}{2} = 1 - \left(\frac{1000}{1000+d}\right)^{2}$$

$$\sqrt{\frac{1}{2}} = \frac{1000}{1000+d} \Rightarrow d = \frac{1000}{\sqrt{\frac{1}{2}}} - 1000 = 414.21$$
2. The random variable $X$ is uniformly distributed between 20 and $z$.

$TVaR_{.80}(X) = 155$.

Determine $k$ so that the standard deviation principle is also equal to 155.

\[
TVaR_{p}(x) = \frac{b + a + p(b - a)}{2} \quad \text{for} \quad p = .80 \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad a = 20 \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad b = z \\
= \frac{z + 20 + (.8)(z - 20)}{2} = 155 \\
\Rightarrow 1.8z + 4 = 310 \quad \Rightarrow z = 170
\]

\[
E(X) = \frac{b + a}{2} = \frac{170 + 20}{2} = 95
\]

\[
Var(X) = \frac{(b - a)^2}{12} = \frac{(150)^2}{12} = 1875
\]

\[
\sigma = \sqrt{1875} = 43.30127019
\]

**Standard Deviation Principle**

\[
\mu + k \sigma = 155
\]

\[
95 + k(43.30127019) = 155
\]

\[
k = \frac{155 - 95}{43.30127019} = 1.38864
\]