1. Amstutz Ant Farm is studying the life expectancy of ants. The farm’s actuary isolates 100 ants and records the following data:

<table>
<thead>
<tr>
<th>Number of Days till Death</th>
<th>Number of Ants Dying</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>35</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

Estimate $\hat{S}(4)$ using the Nelson-Aalen estimator.

\[
\hat{H}(4) = \frac{6}{100} + \frac{10}{94} + \frac{12}{84} + \frac{20}{72} = 0.587017899
\]

\[
\hat{S}(4) = e^{-\hat{H}(4)} = e^{-0.587017899} = 0.556
\]
2. You are given the following frequency distribution:

<table>
<thead>
<tr>
<th>Number of Claims</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.20</td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>0.40</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
</tr>
</tbody>
</table>

You are also given the following severity distribution:

<table>
<thead>
<tr>
<th>Amount of Claim</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.50</td>
</tr>
<tr>
<td>200</td>
<td>0.40</td>
</tr>
<tr>
<td>500</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Calculate $F_S(350)$.

$$F_S(350) = F_S(0) + F_S(100) + F_S(200) + F_S(300)$$

$$F_S(0) = Pr(N=0) = 0.20$$

$$F_S(100) = Pr(N=1) \cdot Pr(X=100) = (0.25)(.5) = 0.125$$

$$F_S(200) = Pr(N=1) \cdot Pr(X=200) + Pr(N=2) \cdot Pr(Both X=100)$$

$$= (0.25)(.4) + (.4)(.5)^2 = 0.20$$

$$F_S(300) = Pr(N=2) \cdot Pr(X=100 \text{ and } X=200) + Pr(N=3) \cdot Pr(All X=100)$$

$$= (.4)(.5)(.4)(2) + (.15)(.5)^3 = 1.7875$$

$$F_S(350) = 0.20 + 0.125 + 0.20 + 1.7875 = 0.70375$$
3. You are given the following frequency distribution:

<table>
<thead>
<tr>
<th>Number of Claims</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.20</td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>0.40</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
</tr>
</tbody>
</table>

You are also given the following severity distribution:

<table>
<thead>
<tr>
<th>Amount of Claim</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.50</td>
</tr>
<tr>
<td>200</td>
<td>0.40</td>
</tr>
<tr>
<td>500</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Calculate the Net Stop Loss premium for an aggregate deductible of 1000.

We can only exceed the deductible if we have

3 claims for 500 each

or 3 claims with 2 for 500

and one of either 100 or 200

Net Stop Loss Premium=

\[(1500 - 1000) \cdot \Pr(N = 3) \cdot \Pr(\text{All } X = 500) \]
\[+ (1200 - 1000) \cdot \Pr(N = 3) \cdot \Pr(\text{Two } X = 500 \text{ and } X = 200) \]
\[+ (1100 - 1000) \cdot \Pr(N = 3) \cdot \Pr(\text{Two } X = 500 \text{ and } X = 100) \]
\[= 500 (0.15)(0.1)^3 + (200)(0.15)(0.1)^2(0.9)(3) \]
\[+ 600(0.15)(0.1)(1.5)(3) = 0.66 \]
4. The number of claims under a five year medical insurance policy is distributed as a Poisson distribution with a mean of 100. The severity of claims is distributed as a Pareto distribution with $\theta = 1000$ and $\alpha = 3$.

Calculate the 90% confidence interval for the expected aggregate claims if aggregate claims are assumed to follow a normal distribution.

\[ E(N) = 100 \quad \text{Var}(N) = 100 \]
\[ E(X) = \frac{\theta}{\alpha - 1} = \frac{1000}{3 - 1} = 500 \]
\[ \text{Var}(X) = \frac{\theta^2 \alpha}{(\alpha - 1)(\alpha - 2)} = \frac{(1000)^2 \cdot (3)}{(2)^2} = 750,000 \]
\[ E(S) = E(N) \cdot E(X) = (100)(500) = 50,000 \]
\[ \text{Var}(S) = E(N) \cdot \text{Var}(X) + \text{Var}(N) [E(X)]^2 \]
\[ = (100)(750,000) + (100)(500)^2 \]
\[ = 100,000,000 \]

90% confidence interval =
\[ E(S) \pm 1.645 \sqrt{\text{Var}(S)} = \]
\[ 50,000 \pm 1.645 \sqrt{100,000,000} \]
\[ = 50,000 \pm 1.645(10,000) \]
\[ = (33,550, 66,450) \]
5. Sahlin Health Insurance Company is providing all students in STAT 479 with a health insurance policy. The health insurance policy provides payment if a person in this class is hospitalized with H1N1.

The following table is a census of the students covered and their probability of claim:

<table>
<thead>
<tr>
<th>Gender</th>
<th>Number of Students</th>
<th>Probability of Claim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>12</td>
<td>0.05</td>
</tr>
<tr>
<td>Male</td>
<td>16</td>
<td>0.10</td>
</tr>
</tbody>
</table>

If a person is hospitalized, claims are distributed as a Gamma distribution with $\alpha = 2$ and $\theta = 2000$ without regard to gender.

Claims are independent.

Calculate $E(S)$ and $Var(S)$.

\[
E(S) = \sum q_i \cdot \mu_i
\]
\[
\]
\[
= 8800
\]

\[
Var(S) = \sum q_i \cdot \sigma_i^2 + q_i (1-q_i) \cdot \mu_i^2
\]
\[
= (12)(.05)(2)(2000^2) + (0.05)(0.95)(4000^2)
\]
\[
+ 16 (1)(2)(2000^2) + (1)(0.9)(4000^2)
\]
\[
= 49,760,000
\]
6. Disability claims are assumed to be distributed uniformly from zero to $\theta$. A sample of four disability claims are drawn and used to estimate $\theta$ by taking the maximum of the sample as the estimate of $\theta$.

The true distribution of disability claims is a uniform distribution from zero to 1000.

Calculate the Mean Square Error of our estimator.

$$\text{MSE} = \text{Var} (\hat{\theta}) + (\text{bias} \hat{\theta})^2$$

$$\text{Var} (\hat{\theta}) = \frac{n \theta^2}{(n+2)(n+1)} = \frac{4(1000)^2}{6 \cdot (5)^2} = 26,666.67$$

$$\text{bias} = (E (\hat{\theta}) - \theta)^2 = \left( \frac{n \theta}{n+1} - \theta \right)^2$$

$$= \left( \frac{4 \cdot 1000}{5} - 1000 \right)^2 = (-200)^2 = 40,000$$

$$\text{MSE} = 26,666.67 + 40,000 = 66,666.67$$
7. Dental claims are distributed exponentially with $\theta = 100$. You want to discretize the claims using a span of 20 using both the Method of Rounding and the Method of Moment Matching where you will match the mean.

Calculate the probability assigned to the range that includes 100 under each method.

Ranges with $h = 20$

$0 - 10$
$10 - 30$
$30 - 50$
$50 - 70$
$70 - 90$
$90 - 110$

Method of Rounding

$$f_5 = F(110) - F(90)$$
$$= (1 - e^{-\frac{110}{100}}) - (1 - e^{-\frac{90}{100}})$$
$$= e^{-0.9} - e^{-1.1} = 0.073698576$$

Method of Moment Matching

$$f_5 = \frac{2E(X \land 100) - E(X \land 80) - E(X \land 120)}{20}$$
$$= \frac{2(100)(1-e^{-\frac{100}{100}}) - (100)(1-e^{-\frac{80}{100}}) - 100(1-e^{-\frac{120}{100}})}{20}$$
$$= 0.073698576$$
8. You are given the following sample data:

\[ X: 5, 6, 8, 10, 10, 10, 13, 17, 20, 21 \]

You are also given:

\[ N = 10 \]
\[ \text{Sum of } X_i = 120 \]
\[ \text{Sum of } X_i^2 = 1724 \]

You complete Hypothesis Testing with:

\[ H_0: \text{The mean is } 10. \]
\[ H_1: \text{The mean is not } 10. \]

Calculate the \( z \) statistic, the critical value(s) assuming a significance level of 30\%, and the \( p \) value. State your conclusion with regard to the Hypothesis Testing.

\[
\bar{X} = \frac{120}{10} = 12
\]

\[
\text{Var}(X) = \frac{\sum (X_i^2) - N(\bar{X}^2)}{N-1} = \frac{1724 - 10(12)^2}{9} = 31.556
\]

\[
Z = \left| \frac{12 - 10}{\sqrt{\frac{31.556}{10}}} \right| = 1.1258799
\]

Critical values = \( \pm 1.036 \)

We reject \( H_0 \) because \( 1.1258799 > 1.036 \)

\( p \) values = \( 2 \left( 1 - \Phi(1.1258799) \right) = 0.24 \)
9. Troup Trucking Company suffered 100 accidents last year. The following table summarizes the amount of losses on those 100 accidents:

<table>
<thead>
<tr>
<th>Amount of Loss</th>
<th>Number of Accidents</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 10,000</td>
<td>15</td>
</tr>
<tr>
<td>10,000 to 25,000</td>
<td>25</td>
</tr>
<tr>
<td>25,000 to 100,000</td>
<td>35</td>
</tr>
<tr>
<td>100,000 to 250,000</td>
<td>20</td>
</tr>
<tr>
<td>Over 250,000</td>
<td>5</td>
</tr>
</tbody>
</table>

Calculate $F_{100}(x)$ for losses between 25,000 and 100,000 using the Ogive.

\[
F(25,000) = \frac{40}{100} = .4
\]

\[
F(100,000) = \frac{75}{100} = .75
\]

Ogive is linear interpolation between these values:

\[
\frac{100,000 - x}{75,000} (0.40) + \frac{x - 25,000}{75,000} (0.75)
\]

\[
= \frac{40,000 - 0.4x + 0.75x - 18,750}{75,000}
\]

\[
= 21,250 + 0.35x
\]

\[
75,000
\]
10. Troup Trucking’s main competitor, Huang Hauling LTD, has the following accident information from last year:

<table>
<thead>
<tr>
<th>Amount of Loss</th>
<th>Number of Accidents</th>
<th>Total Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 10,000</td>
<td>5</td>
<td>30,000</td>
</tr>
<tr>
<td>10,000 to 25,000</td>
<td>8</td>
<td>130,000</td>
</tr>
<tr>
<td>25,000 to 100,000</td>
<td>12</td>
<td>840,000</td>
</tr>
<tr>
<td>100,000 to 250,000</td>
<td>3</td>
<td>460,000</td>
</tr>
<tr>
<td>Over 250,000</td>
<td>2</td>
<td>740,000</td>
</tr>
</tbody>
</table>

Huang Hauling buys truck insurance which will pay claims up to a limit of 100,000 per claim.

Using the distribution of claims from last year, calculate the expected value per payment for the insurance company.

\[
\text{Total Payments} = 30,000 + 130,000 + 840,000 + 3(100,000) + 2(100,000) = 1,500,000
\]

\[
\text{Number of Payments} = 30
\]

\[
\text{Expected Value} = \frac{1,500,000}{30} = 50,000
\]