1. You are given the following random sample from an Exponential distribution:

3 6 8 10 20

Using percentile matching at the 80\textsuperscript{th} percentile, estimate \( \theta \).

\begin{align*}
\bar{x} &= 10 \quad \bar{y} = 20 \\
80\text{th percentile} &= (1 - 0.8)(10) + (0.8)(20) \\
&= 18 \\
F(18) &= 1 - e^{-18/\theta} = 0.80 \\
0.2 &= e^{-18/\theta} \\
\ln(0.2) &= -18/\theta \\
\theta &= \frac{-18}{\ln(0.2)} = 11.18403
\end{align*}
2. You are given the following random sample from a Pareto distribution:

10  40  200  750

Using the method of moments, estimate $\alpha$ and $\theta$.

\[
E(X) = \frac{10 + 40 + 200 + 750}{4} = 250 = \frac{\theta}{\alpha-1}
\]

\[
E(X^2) = \frac{10^2 + 40^2 + 200^2 + 750^2}{4} = 151,050 = \frac{2\theta^2}{(\alpha-1)(\alpha-2)}
\]

\[
\frac{E(X^3)}{[E(X)]^2} = \frac{2\theta^2}{(\alpha-1)(\alpha-2)} \cdot \frac{(\alpha-1)^2}{\theta^2} = \frac{2(\alpha-1)}{\alpha-2} = \frac{151,050}{(250)^2}
\]

\[
2\alpha - 2 = 2.4168(\alpha - 2) = 2.4168\alpha - 4.8336
\]

\[
2,8336 = .4168\alpha
\]

\[
\alpha = 6.79846
\]

\[
\frac{\theta}{\alpha-1} = 250 \Rightarrow \theta = 250(5.79846)
\]

\[
= 1419.61612
\]