1. A two point mixture distribution of $X$ is comprised of the following two distributions:

$X_1$. A uniform distribution where random variable is uniformly distributed between 100 and 500.

$X_2$. An empirical distribution with a mean of 100 and a standard deviation of 200.

The weight for the uniform distribution is 0.6 and the weight for the empirical distribution is 0.4.

Calculate the variance of $X$.

$$E(X) = \frac{1}{500} \int_{100}^{500} x \left( \frac{1}{500} \right) dx = \frac{(500)^2 - (100)^2}{500} = \frac{500 + 100}{2} = 300$$

$$E(X^2) = \frac{1}{500} \int_{100}^{500} x^2 \left( \frac{1}{500} \right) dx = \frac{(500)^3 - (100)^3}{1200} = 103,333.33$$

$a_1 = 0.6$

$X_2$

$$E(X_2) = 100$$

$$E(X_2^2) = \text{Var}(X_2) + [E(X_2)]^2 = (200^2 + 100^2) = 50,000$$

$a_2 = 0.4$

$$E(X) = 0.6 E(X_1) + 0.4 E(X_2) = (0.6)(300) + (0.4)(100) = 220$$

$$E(X^2) = 0.6 E(X_1^2) + 0.4 E(X_2^2) = (0.6)(103,333.33) + (0.4)(50000) = 82,000$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = 82,000 - 220^2 = 33,100$$
2. You are given the following empirical distribution:

\[1 2 3 4 5 6 7 8 9 10\]

\(k_1\) is the value of \(k\) in the standard deviation principle so that the risk measure under the standard deviation principle is equal to \(\text{VaR}_{0.75}(X)\).

\(k_2\) is the value of \(k\) in the standard deviation principle so that the risk measure under the standard deviation principle is equal to \(\text{TVaR}_{0.75}(X)\).

Calculate \(k_2 - k_1\).

\[
\begin{align*}
E(x) &= \frac{1+2+3+4+5+6+7+8+9+10}{10} = 5.5 \\
E(x^2) &= \frac{1+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2}{10} = 38.5 \\
\text{Var}(x) &= 38.5 - (5.5)^2 = 8.25 \\
\sigma &= \sqrt{8.25} = 2.8723
\end{align*}
\]

Standard deviation principle: \(\mu + k\sigma\)

\(\text{VaR}_{0.75}(x) = 8\)

\[5.5 + k_1(2.8723) = 8 \Rightarrow k_1 = 0.8704\]

\(\text{TVaR}_{0.75}(x) = \frac{9+10}{2} = 9.5\)

\[5.5 + k_2(2.8723) = 9.5 \Rightarrow k_2 = 1.3926\]

\[k_2 - k_1 = 1.3926 - 0.8704 = 0.5222\]