1. Cao Casualty Company sells stop loss insurance which covers the Beckley Family medical bills. The stop loss insurance pays for aggregate claims in excess of 1000.

There are three members of the Beckley family. The distribution of loss for each member of the family is:

<table>
<thead>
<tr>
<th>Amount of Loss</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>1000</td>
<td>0.3</td>
</tr>
<tr>
<td>2000</td>
<td>0.2</td>
</tr>
<tr>
<td>3000</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Calculate the net stop loss premium.

**Solutions:**

\[
Net \ Stop \ Loss \ Premium = E[S] - E[S \wedge 1000]
\]

\[
E[S] = E[N]E[X]
\]

\[
E[N] = 3; E[X] = (0)(0.4) + (1000)(0.3) + (2000)(0.2) + (3000)(0.1) = 1000
\]

\[
E[S] = E[N]E[X] = (3)(1000) = 3000
\]

*If there are 1 claim or more, then \(X \wedge 1000\) results in a payment of 1000.* \(Pr(\text{zero claims}) = (0.4)^3.\)

\[
E[S \wedge 1000] = (0)(0.4)^3 + (1000)[1 - (0.4)^3] = 936
\]

\[
E[S] - E[S \wedge 1000] = 3000 - 936 = 2064
\]
2. The number of claims for each insured under dental insurance is distributed as a Zero-Modified Geometric Distribution. The probability of having zero losses is 20% and $E[N] = 2$.

Losses are distributed as a Pareto distribution with $\theta = 500$ and $\alpha = 3$.

Zhang Dental Corporation issues a policy that covers 10,000 independent insureds. The premium for this dental insurance is equal to the expected value of the aggregate claims for the policy plus one standard deviation.

Calculate the premium.

**Solution:**

$$E[X] = \frac{\theta}{\alpha - 1} = \frac{500}{3 - 1} = 250; \ E[S] = E^M[N]E[X] = (2)(250) = 500; \ E[\text{Portfolio}] = (10,000)(500) = 5,000,000$$

$$E^M[N] = 2 = (1 - p_0^M)E^X = (1 - 0.2)(\beta + 1) \implies \beta = \frac{2}{(1 - 0.2)} - 1 = 1.5$$

$$E^X = 1 + \beta = 1 + 1.5 = 2.5; \ Var^X = (\beta)(1 + \beta) = (1.5)(2.5) = 3.75$$

$$Var^M[N] = (1 - p_0^M)(Var^X) + (1 - p_0^M)(p_0^M)(E^X)^2 = (0.8)(3.75) + (0.8)(0.2)(2.5)^2 = 4$$

$$Var[X] = \frac{\alpha \theta^2}{(\alpha - 1)^2 (\alpha - 2)(\alpha - 3)} = \frac{(3)(500)^2}{(2)^2 (1)} = 187,500$$

$$Var(S) = E[N]Var[X] + Var[N](E[X])^2 = (2)(187,500) + (4)(250)^2 = 625,000$$

$$Var(\text{Portfolio}) = (10,000)(625,000) = 6,250,000,000$$

$$\text{Premium} = E[\text{Portfolio}] + \sqrt{Var(\text{Portfolio})} = 5,000,000 + \sqrt{6,250,000,000} = 5,079,056.94$$