1. Claims for medical insurance coverage is distributed as an exponential distribution with $\theta = 500$.

Siyun creates a discrete distribution of claims by using the Method of Mass Dispersal with a span of 50.

Yuxi creates a discrete distribution of claims by using the Method of Moment Matching such that she matches the mean with a span of 50.

Calculate the probability assigned to the claim amount of 500 by Siyun and by Yuxi.

Solution:

Method of Mass Dispersal - Siyun

$$F(525) - F(475) = \left[ 1 - e^{-\frac{525}{500}} \right] - \left[ 1 - e^{-\frac{475}{500}} \right] = e^{-\frac{475}{500}} - e^{-\frac{525}{500}} = 0.03680327$$

Method of Moment Matching - Yuxi

$$\frac{2E[X \wedge 500] - E[X \wedge 450] - E[X \wedge 550]}{50} =$$

$$\frac{2\left[ 500 \left( 1 - e^{-\frac{500}{500}} \right) \right] - \left[ 500 \left( 1 - e^{-\frac{450}{500}} \right) \right] - \left[ 500 \left( 1 - e^{-\frac{550}{500}} \right) \right]}{50} = 0.0368186$$
2. The number of claim payments follows a Poisson distribution with $\lambda = 2.5$.

The amount of each claim has the following distribution: $f_X(50) = 0.6$ and $f_X(100) = 0.4$.

Calculate $f_S(200)$.

**Solution:**

There are the following combinations that will total 200

- Two claims where each claim is 100.
- Three claims where two claims are 50 and one claim is 100.
- Four claims where all four claims are for 50.

$$f_S(200) = \Pr(N = 2)(\Pr(X = 100))^2 + \Pr(N = 3)(\Pr(X = 50))^2 \Pr(X = 100)(3) + \Pr(N = 4)(\Pr(X = 50))^4 =$$

$$= \frac{e^{-2.5}(2.5)^2}{2!}(0.4)^3 + \frac{e^{-2.5}(2.5)^3}{3!}(0.6)^2(0.4)(3) + \frac{e^{-2.5}(2.5)^4}{4!}(0.6)^2 = 0.150703$$