1. Let \( X \) be the random variable representing the loss under an insurance policy.

You are given that the cumulative distribution function for \( X \) is:

\[
F(x) = \begin{cases} 
\frac{x^2}{625} & \text{for } 0 \leq x \leq 25 \\
1 & \text{for } x > 25
\end{cases}
\]

The premium for the policy is the larger of:

- The mean plus 50% of the standard deviation; or
- The 100\( p \)^{th} percentile.

Determine \( p \) such that the two methods produce the same premium.

Solutions:

\[
f(x) = F'(x) = \frac{2x}{625}
\]

\[
E[X] = \int_0^{25} x \cdot f(x) \, dx = \int_0^{25} x \cdot \frac{2x}{625} \, dx = \int_0^{25} \frac{2x^2}{625} \, dx = \frac{2x^3}{1875} \bigg|_0^{25} = 16.6666
\]

\[
E[X^2] = \int_0^{25} x^2 \cdot f(x) \, dx = \int_0^{25} x^2 \cdot \frac{2x}{625} \, dx = \int_0^{25} \frac{2x^3}{625} \, dx = \frac{x^4}{1250} \bigg|_0^{25} = 312.5
\]

\[
Var[X] = E[X^2] - (E[X])^2 = 312.5 - (16.6666)^2 = 34.722222
\]

\[
\sigma = \sqrt{Var[X]} = \sqrt{34.722222} = 5.8925565
\]

Premium = \( E[X] + 0.5\sigma = 16.6666 + 0.5(5.8925565) = 19.61294492 = \pi_p \)

\[
F(\pi_p) = p = \frac{(19.61294492)^2}{625} = 0.6155
\]
2. The Li Insurance Company provides health insurance. The company receives claims from 5000 males and from 4000 females. Each claim is independent of any other claim.

The amount of the claims for males are distributed as a Gamma distribution with \( \alpha = 5 \) and \( \theta = 800 \).

The amount of the claims for females are distributed as a Pareto distribution with \( \alpha = 5 \) and \( \theta = 10,000 \).

Calculate the probability that the total aggregate claims (the total amount paid for all 9000 claims) exceeds 30.3 million. Assume that total aggregate claims are distributed as a normal distribution.

Solutions:

\[
E[X_M] = \theta(\alpha + 1 - 1) = 800(5) = 4000 \\
E[X_M^2] = \theta^2(\alpha + 2 - 1)(\alpha + 1 - 1) = 800^2(6)(5) = 19,200,000 \\
Var[X_M] = E[X_M^2] - [E[X_M]]^2 = 19,200,000 - 4000^2 = 3,200,000
\]

\[
E[X_F] = \theta / (\alpha - 1) = 10,000 / 4 = 2500 \\
E[X_F^2] = 2\theta^2 / (\alpha - 1)(\alpha - 2) = 2(10,000)^2 / [4 \cdot 3] = 16,666,666.67 \\
Var[X_F] = E[X_F^2] - [E[X_F]]^2 = 16,666,666.67 - 2500^2 = 10,416,666.67
\]

\[
S = X_{M_1} + \ldots + X_{M_{5000}} + X_{F_1} + \ldots + X_{F_{4000}} \\
E[S] = E[X_{M_1}] + \ldots + E[X_{M_{5000}}] + E[X_{F_1}] + \ldots + E[X_{F_{4000}}] = 5000(4000) + 4000(2500) = 30,000,000 \\
Var[S] = Var[X_{M_1}] + \ldots + Var[X_{M_{5000}}] + Var[X_{F_1}] + \ldots + Var[X_{F_{4000}}] = 5000(3,200,000) + 4000(10,416,666.67) = 57,666,666,680
\]

\[
Pr(S > 30,300,000) = Pr\left( Z > \frac{30,300,000 - 30,000,000}{\sqrt{57,666,666,680}} \right) = Pr(Z > 1.25) = 1 - \Phi(1.25) = 1 - 0.8944 = 0.1056
\]