1. The Zhu Insurance Company provides dental insurance. The following is a sample of claims from Zhu:

25  30  45  60  120

Zhu’s chief actuary Wenchu believes that dental claims are distributed as a Gamma distribution. She estimates the parameters of the Gamma distribution using the Method of Moment Matching.

Using the Gamma distribution with the estimated parameters, determine the most likely value of the next claim that Zhu receives.

\[
E[x] = \frac{25 + 30 + 45 + 60 + 120}{5} = \frac{280}{5} = 56
\]

\[
\text{Var}(x) = E[x^2] - (E[x])^2
\]

\[
= \frac{25^2 + 30^2 + 45^2 + 60^2 + 120^2}{5} - (56)^2 = 1174
\]

Gamma

\[
E(x) = a\theta
\]

\[
\text{Var}(x) = a\theta^2
\]

\[
\frac{\text{Var}(x)}{E(x)} = \frac{a\theta^2}{a\theta} = \theta = \frac{1174}{56} = 20.9642
\]

\[
a = \frac{56}{20.9642} = 2.6712
\]

Most likely value = Mode = \[\theta(a-1) = 20.9642(2.6712) = 35.04\]
2. The Zhu Insurance Company provides dental insurance. The following is a sample of claims from Zhu:

\[ 25 \ 30 \ 45 \ 60 \ 120 \]

Bolun, as consulting actuary retained by Zhu, believes that claims are distributed as an exponential distribution. Further, he decides to estimate the parameter for the exponential distribution using the percentile matching estimate at a percentile of 80%.

Calculate the \( \hat{\theta} \), the estimated parameter of the exponential distribution as calculated by Bolun.

\[
\begin{align*}
(\bar{x})(n+1) &= (4.8)(6) = 4.8 \\
\Rightarrow (60)(0.2) + 120(0.8) &= 96 \\
F(96) &= 0.8 \\
1 - e^{-\frac{96}{\theta}} &= 0.8 \\
e^{-\frac{96}{\theta}} &= 0.2 \\
\frac{-96}{\theta} &= \ln(0.2) \\
\theta &= \frac{-96}{\ln(0.2)} = 59.65
\end{align*}
\]