1. You are given the following sample of claims that were the following amounts:

   100  100  200  300  450

You use the Kolmogorov-Smirnov test at a 90% significance level to test the following hypothesis:

   \( H_0 \): The data is from an exponential distribution with a mean of 200

   \( H_1 \): The data is not from an exponential distribution with a mean of 200

   a. Find the Kolmogorov-Smirnov Test Statistic.
   b. Determine the critical value for this Test.
   c. State your conclusion.

**Solution:**

\[
\begin{array}{|c|c|c|c|c|}
\hline
x & F_n^-(x) & F_n(x) & F_n^+(x) & \text{Greatest Absolute Value} \\
\hline
100 & 0 & 0.4 & 1 - e^{-\frac{100}{200}} = 0.39346934 & 0.39346934 \\
200 & 0.4 & 0.6 & 1 - e^{-\frac{200}{200}} = 0.632120569 & 0.232120559 \\
300 & 0.6 & 0.8 & 1 - e^{-\frac{300}{200}} = 0.77686984 & 0.17686984 \\
450 & 0.8 & 1.0 & 1 - e^{-\frac{450}{200}} = 0.894600775 & 0.105399225 \\
\hline
\end{array}
\]

\( D^* = \text{Greatest Difference} = 0.3935 \)

**Critical value**

\[
\frac{1.22}{\sqrt{5}} = 0.5456
\]

**Conclusion.**

We do not reject as 0.5456 > 0.3934
2. During 2013, Chen Dental Insurance Company paid 5000 claims which were distributed as follows:

<table>
<thead>
<tr>
<th>Amount of Payment</th>
<th>Number of Payments</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 1000</td>
<td>950</td>
</tr>
<tr>
<td>1000 - 4000</td>
<td>1750</td>
</tr>
<tr>
<td>4000 - 8000</td>
<td>1300</td>
</tr>
<tr>
<td>8000+</td>
<td>1000</td>
</tr>
</tbody>
</table>

Jacqui decides to test the following hypothesis using a Chi Square Test at a 95% significance level:

$H_0$: The data is from an Exponential distribution with $\theta = 5000$.

$H_1$: The data is not from an Exponential distribution with $\theta = 5000$.

a. Determine the test statistic $\chi^2$.

b. Determine the critical value for this test.

c. State your conclusion.

Solution:

<table>
<thead>
<tr>
<th>Range</th>
<th>$O_j$</th>
<th>$E_j$</th>
<th>$(E_j - O_j)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1000</td>
<td>950</td>
<td>$5000[F(1000) - F(0)] = 906.346$</td>
<td>2.1026</td>
</tr>
<tr>
<td>1000-4000</td>
<td>1750</td>
<td>$5000[F(4000) - F(1000)] = 1847.009$</td>
<td>5.0951</td>
</tr>
<tr>
<td>4000-8000</td>
<td>1300</td>
<td>$5000[F(8000) - F(4000)] = 1237.162$</td>
<td>3.1917</td>
</tr>
<tr>
<td>8000+</td>
<td>1000</td>
<td>$5000[1 - F(8000)] = 1009.483$</td>
<td>0.0891</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>5000</td>
<td><strong>Total</strong></td>
<td><strong>10.41785</strong></td>
</tr>
</tbody>
</table>

Critical value

Degrees of Freedom $= 4 - 1 = 3$ so $c=7.815$

Conclusion.

Reject $H_0$ since $7.815<10.478$
3. Your company has recently started issuing a hospitalization policy with an upper limit of 5000. The first five claims that have been paid are as follows:

\[ 900 \quad 4000 \quad 4600 \quad 5000 \quad 5000 \]

Liren, the chief actuary for your company is concerned because to determine the premiums for this policy, the losses were modeled as an exponential with a mean of 2500. She asks you to test the following hypothesis using the Likelihood Ratio Test at a 90% significance level:

- **H₀**: The data is from an Exponential distribution with \( \theta = 2500 \).
- **H₁**: The data is from an Exponential distribution.

**a.** Determine the test statistic.
**b.** Determine the critical value for this test.
**c.** State your conclusion.

**Solution:**

\[
L(\theta) = \frac{e^{-900/\theta}}{\theta} \cdot \frac{e^{-4000/\theta}}{\theta} \cdot \frac{e^{-4600/\theta}}{\theta} \cdot \frac{e^{-5000/\theta}}{\theta} \cdot \frac{e^{-5000/\theta}}{\theta} = \frac{e^{-19400/\theta}}{\theta^3}
\]

\[
L(\theta_0) = \frac{e^{-19400/2500}}{2500^3}
\]

\[
\theta = \frac{900 + 4000 + 4600 + 5000 + 5000}{3} = 6500
\]

\[
L(\theta_1) = \frac{e^{-19400/6500}}{6500^3}
\]

\[
T = 2 \left[ \ln \left( \frac{e^{-19400/6500}}{6500^3} \right) - \ln \left( \frac{e^{-19400/2500}}{2500^3} \right) \right] = 3.867
\]

Critical Value \(\Rightarrow\) Degrees of Freedom = 2 - 1 = 1 \(\Rightarrow\) 2.706

Conclusion \(\Rightarrow\) Reject \(H₀\) since 3.867 > 2.706
4. **State the Principle of Parsimony.**

**Solution:**

Use a simple model unless there is overwhelming evidence to the contrary.

5. You have the following sample of claims:

1700 3000 5800 10,000 15,000

You believe that the underlying distribution is a Pareto distribution with $\alpha = 5$.

You want to graphic evaluate this assumption.

Calculate $D'(3000)$.

**THIS QUESTION CANNOT BE DONE AS WRITTEN.**