(1) Consider \( f(x, y) = x^4 + y^4 - 4xy + 1 \). Show that \( f \) has a local minimum at \((1, 1)\) and \((-1, -1)\) and that \((0, 0)\) is a saddle point of \( f \).

(2) Find the extreme values of the function \( f(x, y) = x^2 + 2y^2 \) on the circle \( x^2 + y^2 = 1 \). Answer: \( f(0, \pm 1) = 2 \) is the maximum, \( f(\pm 1, 0) = 1 \) is the minimum value.
(3) Find the area of one loop of the rose \( r = \cos(2\theta) \) sketched below. Answer: \( \frac{\pi}{8} \)

(4) Find the value of the integral \( I = \int_{y^2}^{\sqrt{2}} \int_{y^2}^{2} y e^{x^2} \, dx \, dy \) by interchanging the order of integration. Answer:

\[
I = \int_{0}^{\sqrt{2}} \int_{0}^{\sqrt{x}} y e^{x^2} \, dy \, dx = \frac{1}{4}(e^4 - 1).
\]

See also similar problems on page: 996 (15.3(#49−54)).
(5) Use the midpoint rule with \( m = n = 2 \) to approximate
\[
\iint_{R} (x^2 - 1) y \, dA
\]
where \( R \) is the region \( \{(x, y) : 0 \leq x \leq 4, \ 2 \leq y \leq 4\} \).
Answer: 96

(6) Let \( R \) be the region in the first quadrant bounded by \( x = 0, \ x - y = 0, \ x^2 + y^2 = 9 \) and \( x + y = 6 \). Evaluate
\[
\iint_{R} \frac{x + y}{x^2 + y^2} \, dA.
\]
Answer: \( \frac{3}{2} \pi - 3 \)
(7) Find the center of mass \((\bar{x}, \bar{y})\) of the semicircular lamina described by \(\{(x, y) : x^2 + y^2 \leq a^2, y \geq 0\}\) if its density at the point \((x, y)\) is \(\rho(x, y) = \sqrt{x^2 + y^2}\).

Answer: \(\bar{x} = 0, \quad \bar{y} = \frac{3a}{2\pi}\)

(8) Find the area of the region described by the intersection of two disks bounded by \(x^2 + y^2 = x\) and \(x^2 + y^2 = y\).

Answer: \(\frac{\pi}{8} - \frac{1}{4}\)
(9) Find $a, b, c, d, e, f, g, h$ so that
\[
\int_0^1 \int_0^y \int_{\sqrt{\frac{y}{x}}}^1 F(x, y, z) \, dx \, dy \, dz = \int_0^1 \int_a^b \int_c^d F(x, y, z) \, dy \, dx \, dz = \int_0^1 \int_e^f \int_g^h F(x, y, z) \, dz \, dy \, dx
\]
Answer: $a = \sqrt{z}$, $b = 1$, $c = z$, $d = x^2$, $e = 0$, $f = x^2$, $g = 0$, $h = y$.

(10) Find the volume the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ below the sphere $x^2 + y^2 + z^2 = 2z$.
Answer: $\pi$
(11) Let $T$ be the solid region in the first octant that is bounded by the planes $y = 2$, $x = 0$, $y = 2x$, $z = 0$, and $z = 2y$. What is the value of the triple integral $\iiint_T x \, dV$?

Answer: $1$

(12) Let $c$ be a constant such that $0 < c < \pi/2$ or $\pi/2 < c < \pi$. Show that the equation of the surface $\phi = c$ converted to rectangular coordinates becomes $z = \cot(c)\sqrt{x^2 + y^2}$. 
(13) A lamina $L$ occupies the triangular region in the $xy$–plane with vertices $(0, 0)$, $(0, 3)$ and $(3, 3)$. If the mass density at $(x, y)$ is $\rho(x, y) = x + 2y$, then show that the $y$–coordinate of the center of mass of $L$ is equal to $\frac{9}{4}$.

(14) Let $E$ be the solid region enclosed by the paraboloid $z = x^2 + y^2$ and the plane $2x + z = 4$. Find the triple integral in cylindrical coordinates that gives the volume $V(E)$ of solid $E$.

Answer:

$$V(E) = \int_{0}^{2\pi} \int_{0}^{\cos \theta + \sqrt{4 + \cos^2 \theta}} \int_{\sqrt{4 - 2r \cos \theta}}^{4 - 2r \cos \theta} r \, dz \, dr \, d\theta$$
(15) Convert
\[ \int_{0}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{\sqrt{x^2+y^2}} x y z \, dz \, dy \, dx \]
to spherical coordinates.
Answer:
\[ \int_{-\pi/2}^{\pi/2} \int_{-\pi/4}^{\pi/4} \int_{0}^{3/\sin \phi} \rho^5 \cos \phi \sin^3 \phi \cos \sin \theta \, d\rho \, d\phi \, d\theta \]

(16) What is the value of
\[ \int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2} \, dz \, dx \, dy \, dy \, dx \]
Answer: \( \frac{64\pi}{9} \).
(17) Evaluate \( \iiint_E \sqrt{x^2 + z^2} \, dV \), where \( E \) is the region bounded by the paraboloid \( y = x^2 + z^2 \) and the plane \( y = 4 \).
Answer: \( \frac{128\pi}{15} \).

(18) Find the surface area of the part of the paraboloid \( z = x^2 + y^2 \) that lies under the plane \( z = 9 \).
Answer: \( \frac{\pi}{6}(37\sqrt{37} - 1) \).
(19) Find $K$, $L$, $M$, and $N$ so that

\[
\int_0^2 \int_{1-(x^2/4)}^{1-(x/2)} f(x, y) \, dy \, dx = \int_K^L \int_M^N f(x, y) \, dx \, dy.
\]

Answer: $K = 0$, $L = 1$, $M = 2 - 2y$, $N = 2\sqrt{1-y}$.

(20) Write the double integral in polar coordinates representing the area of the planar region bounded on the right by $x^2 + y^2 = 2$ and bounded on the left by $x = 1$.

Answer: $\int_{-\pi/4}^{\pi/4} \int_{\sec \theta}^{\sqrt{2}} r \, dr \, d\theta = \frac{\pi}{2} - 1$. 