PROBLEM SET #3

1. **Page 149:** # 1, 6, 18.

2. A particle travels along the circular helix \( \mathbf{c}(t) = (2\cos t, 2\sin t, 1 - 4t) \) until it flies off on a tangent when \( t = 0 \). If it flies off at a speed of 2 units/sec, where is the particle at \( t = 5 \)?

3. Let \( f : \mathbb{R}^2 \to \mathbb{R} \) and \( g : \mathbb{R}^2 \to \mathbb{R} \) be differentiable, prove that \( \nabla (fg) = f \nabla g + g \nabla f \).

4. If \( w = \sqrt{x^2 + y^2} \) with \( x = 3t + 8 \) and \( y = 2t \), compute \( \frac{dw}{dt} \) at \( t = 0 \).

5. If \( w = \sqrt{x^2 + y^2} \) with \( x = st^2 + 2 \) and \( y = s - 3t \), compute \( \frac{\partial w}{\partial t} \) at \( (s,t) = (1,0) \).

6. If \( g(x, y, z) = (x, x + y, x^2 + z, z) \) and \( f(x_1, x_2, x_3, x_4) = (x_1^3 + x_3, x_2^2 - x_4) \), then compute \( D(f \circ g)(1,1,1) \).

7. Consider the equation below:

\[
x^3 + y^2 - xz^2 = 4x + 1 \quad (\ast)
\]

(a) If \( x = x(y,z) \) is defined implicitly by \( \ast \), compute \( \frac{\partial x}{\partial y} \).

(b) If \( y = y(x,z) \) is defined implicitly by \( \ast \), compute \( \frac{\partial y}{\partial x} \).

(c) If \( z = z(x,y) \) is defined implicitly by \( \ast \), compute \( \nabla z \).

8. Let \( f : \mathbb{R} \to \mathbb{R} \) be a \( C^1 \) function and let \( w(x,y) = f \left( \frac{x + y}{x - y} \right) \). Show that \( w \) satisfies this partial differential equation:

\[
x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} = 0.
\]

9. **Page 171:** # 2(a), 4(a), 5(a), 9.

10. If \( f(x, y, z) = x^2y + xe^z \) and \( \mathbf{c}(t) = (t^2 + t, \frac{1}{t}, 2t - 1) \), compute

(a) the rate of change of \( f \) along the path \( \mathbf{c} \) at \( t = \frac{1}{2} \).

(b) the directional derivative of \( f \) in the direction of the tangent to the path \( \mathbf{c} \) at \( t = \frac{1}{2} \).